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**Rheology of
building materials**

Anders Nielsen

**National Swedish
Building Research**



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I skriften, som är skriven på engelska, ges en översikt över hur byggnadsmaterialen deformeras under långtidslast. En deformationsbeskrivning gemensam för alla material anges. De material som har behandlats är betong, gasbetong, trä, träfiberskivor, spånskivor och plywood samt i mindre omfattning plast och aluminium. Inflytandet av tid, spänning, temperatur och fukt på deformationen diskuteras.

De flesta byggnadsmaterial uppnår icke sina slutliga deformationer omedelbart efter det att lasten är påförd, som det ofta implicit förutsätts vid beräkningarna. Deformationerna fortsätter att växa efter lastpåläggningen och kan öka under flera år. Detta kan leda till oönskade nedböjningar och förspänningsförluster. Å andra sidan ger långtidsdeformationer upphov till spänningsrelaxation. Detta har säkert räddat många konstruktioner med spänningskoncentrationer från allvarlig sprickbildning.

Materialens tidsberoende deformationer beskrivs i vetenskapsgrenen reologi. Speciellt för byggnadsmaterial kan följande definition uppställas:

Byggnadsmaterialreologi behandlar beskrivning av och förklaring till byggnadsmaterialens av yttre last, fukt- och temperaturväxlingar orsakade tidsberoende deformationer.

De fenomen som omfattas av denna definition är, *krypning*, ökning av deformationen under konstant spänning, *relaxation*, minskning av spänningen under konstant deformation, *temperaturbetingade rörelser* samt *krypning* och *svällning*, vilka är rent fuktbetingade. I rapporten behandlas endast krypning, eftersom det finns användbara omräkningsformler mellan krypning och relaxation (BIL. D), och eftersom stor-

leksordningen av de rena fukt- och temperaturbetingade rörelserna kan anses välkända. Emellertid behandlas samverkan mellan fukt- och temperaturbetingade rörelser och krypning. En ytterligare orsak till att behandla krypning i stället för relaxation är att de flesta försök på detta fält är utförda som krypförsök.

I rapporten ges en *fenomenologisk beskrivning* av krypning i enaxligt spänningstillstånd. Fysikaliska förklaringar till varför materialen kryper har endast skisserats. Detta fenomen bör studeras särskilt med användning av grundläggande fysik och kemi.

Begreppsbestämningar

Vid analys av deformationsproblem är det nödvändigt att särskilja de olika deformationstyperna (töjningstyperna) (se FIG. 1) momentant elastisk, försenat elastisk och viskös deformation. Den fjärde deformationstypen, den plastiska, eller momentant irreversibla deformationen, visar sig vara utan betydelse vid de spänningsnivåer som har analyserats. Den har därför ej medtagits i FIG. 1. Kurvan OA kallas *krypkurvan*.

Vid ett försök med upprepad på- och avlastning ser man att den första krypkurvan ger större utslag än såväl återhämtningskurvan som nästa krypkurva. Den första krypkurvan benämnes *jungfrukurvan*.

Krypningens storlek är naturligtvis beroende av materialgrupp, materialtyp och kvalitet. Därutöver inverkar tid, spänning, temperaturnivå, jämviktsfuktnivå och varierande temperatur- och fuktnivå på krypningen. Dessa faktorer inflytande illustreras kvalitativt av FIG. 2. Nedan kommenteras var och en av dessa faktorer.

Tiden

Bland de många empiriska uttryck för

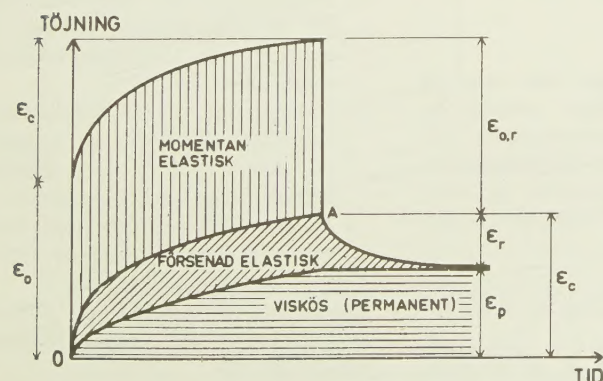


FIG. 1 Definition av töjningskomponenter under pålastning och avlastning. ϵ_0 är momentan töjning, ϵ_e kryptöjning, $\epsilon_{0,r}$ momentan återhämtning, ϵ_r tidsberoende återhämtning och ϵ_p är permanent töjning.

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Nyckelord:

reologi, byggnadsmaterial, långtidsdeformation (fukt, temperatur), krypning, elasticitetsmodul

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jungfrukrypningens, ϵ_c , beroende av tiden, t , (FIG. 2A), visar sig potensfunktionen

$$\epsilon_c(t) = A \cdot t^b \quad (1)$$

mycket användbar. A och b är konstanter; $b < 1$. I ett log-log diagram är ekv. (1) en rät linje med lutningen b och ordinatan $\log A$ för $t = 1$. Empiriska försök visar att potensfunktionen kan beskriva jungfrukrypningen hos de flesta byggnadsmaterial i klimatisk jämvikt vid såväl tryck, drag som böjning. Dessutom kan den användas som approximation för krypningen hos material utsatta för periodiska klimatväxlingar, som tex utomhuskonstruktioner. Konstanten b är till synes oberoende av spänning, temperatur och fukt. Dessa faktors inflytande avspeglas i konstanten A . (Notera att konstanten b är beroende av det sätt på vilket momenttöjningen bestäms.)

Spänningen

Spänningens, σ , inflytande på jungfrukrypningen (FIG. 2B) kan mest generellt uttryckas genom funktionen

$$\epsilon_c = K_1 \sinh(K_2 \sigma) \quad (2)$$

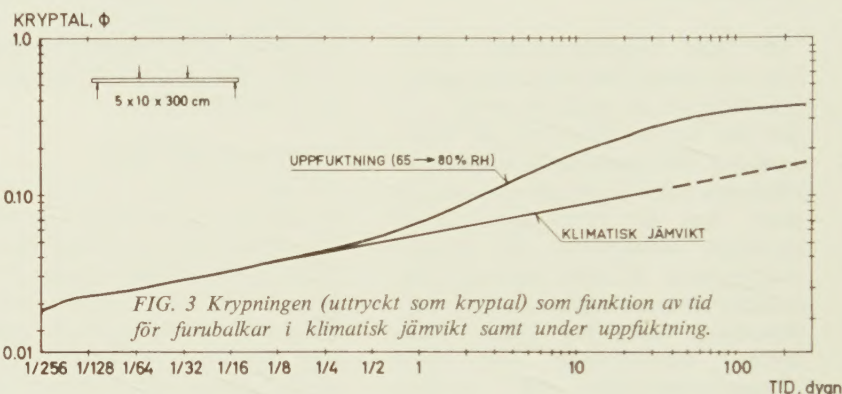
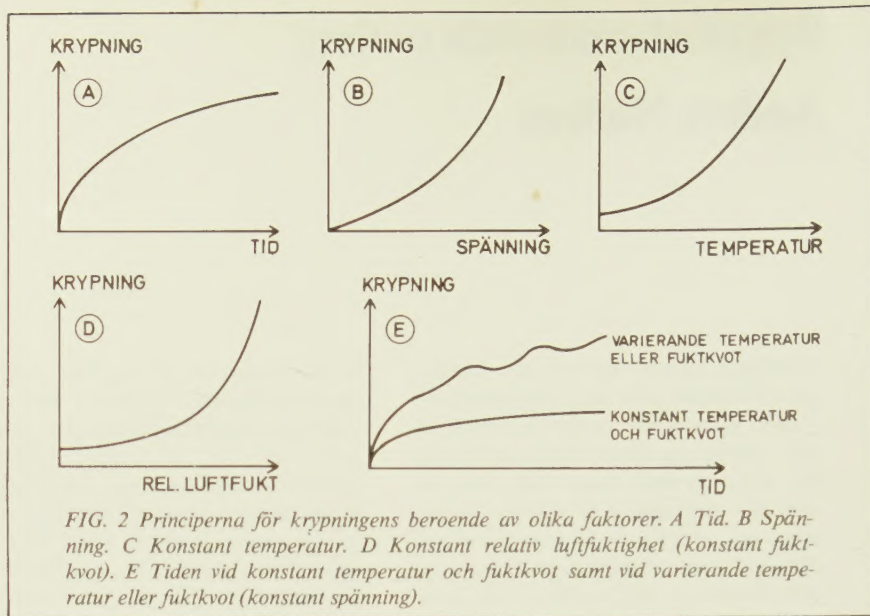
K_1 och K_2 är konstanter. Anpassad på försöksresultat kan ekv. (2) vanligtvis anses vara linjär upp till ca 30 % av brottlasten.

Spänningen påverkar de olika töjningskomponenterna olika. Det förefaller som om momentantöjningen och den försenat elastiska töjningen är linjärt beroende av spänningen även vid mycket höga spänningsnivåer. Det uppställs som en arbetshypotes, att den försenat elastiska töjningen utgör en bestämd bräddel av momentantöjningen för ett givet material i jämvikt i ett bestämt klimat. Detta betyder att den olinjära ökningen av krypningen med spänningen orsakas av att den viskösa komponenten växer olinjärt. Denna hypotes bör undersökas vid framtida forskningsprojekt.

Boltzmanns superpositionsprincip utsäger, tillämpad på krypning, att totalkrypningen till följd av växlande last kan beräknas som summan av de krynningar, som varje laststeg orsakar, avlastningar räknade som negativa krynningar. Denna princip har behandlats och det påpekas att förutsättningen för att den skall kunna användas på byggnadsmaterial är, att man skiljer på försenat elastisk och viskös töjning och använder principen på varje töjningskomponent för sig.

Klimatet

Temperatur och fukt påverkar krypningen på två sätt. För det första kryper materialen mer vid jämvikt vid en högre temperatur eller luftfuktighet än vid en lägre (jfr FIG. 2 C & D). För det andra ökar krypningen också om temperatur och jämviktsfuktförhållanden ändras under det att lasten är



pålagd (jfr FIG. 2 E). Ett exempel ses i FIG. 3 som visar krypning av träbalkar dels vid jämvikt med 65 % relativ fuktighet (RH) och dels vid uppfuktning från jämvikt med 65 % RH till jämvikt med 80 % RH. När uppfuktningen slutar (efter ca 30 dagar) upphör även ökningen av kryphastigheten och man får en S-form i log-log-diagrammet. I rapporten återupplivas den tidigare av Pickett uppställda hypotesen, att de inre spänningar som uppstår under klimatändringen är en huvudorsak till ökningen.

Vissa tumregler kan ges om hur klimatändringar påverkar krypningen. Dock kan man säga att med vårt nuvarande vetande är det ej möjligt att ge en allmän kvantitativ beräkningsmetod för dessa fenomen.

Krypmodul

I statiska beräkningar kan man ta hänsyn till krypningen genom en reduktion av elasticitetsmodulen E_0 . Härvid får man en "långtidselasticitetsmodul" eller krypmodul E_L . Vid reduktionen används kryptalet Φ definierat som

$$\Phi(t) = \epsilon_c(t)/\epsilon_0 \quad (3)$$

Man får med användning av ekv. (1)

$$E_L = E_0/(1 + \Phi_y \cdot t^b) \quad (4)$$

där t räknas i år och Φ_y är kryptalet

efter ett år. Φ_y och b kan relativt lätt utvärderas från givna jungfrukrypkurvor genom uppritning i log-log-diagram. Exempel på E_0 , Φ_y och b är anförda i rapporten för olika material. Temperatur, fukttillstånd och pålastningssätt måste nog specificeras. De rapporterade Φ_y -värdena varierar från 0.2 till 12.0, b -värdena från 0.1 till 0.4 med ett enskilt värde på 0.7.

Framtida forskning

Analyserna i rapporten visar att det är nödvändigt att nå fram till en standardiserad metod för bestämning av krypning, så att den osäkerhet som finns i bestämningen av ϵ_0 och därmed E_0 kan elimineras.

De egentliga forskningsarbetena på området kan delas i två målgrupper.

- Framtagning av empiriska regler för klimatändringars betydelse för krypning och för superpositionsberäkningar.
- Klargörande av de fundamentala kemiska och fysikaliska orsakerna till krypning för att förbättra egenskaperna och förståelsen av materialen och därigenom kunna använda dem på ett bättre sätt.

Rapporten är baserad på litteraturstudier och på egna krypförsök på gasbetong och trä.

This document is a survey of the long-term deformations of building materials. A description of these deformations, common to all the materials, is given. The following materials are considered: concrete, cellular concrete, wood, fiber-board, particle board, plywood, and to a lesser extent polymers and aluminium. The influences of time, stress, temperature and moisture are discussed.

Most building materials do not reach their final deformations immediately after the application of a load, although this is often assumed in design practice. The deformation increases with time, a process which may continue for several years. This may lead to undesired deflections, loss of prestress and crack-formation. On the other hand, long-term deformation causes stress relaxation. Structures with stress concentrations may thereby escape severe crack formation.

Rheology is the science of time dependent deformations. For building materials it can be defined as follows:

Rheology of building materials deals with the description and the explanation of the time dependent deformations caused by mechanical load and changes in moisture content and temperature.

The phenomena covered by this definition are; *creep*, the time dependent deformation of a material under a constant stress, *relaxation*, the time dependent decrease of stress under a constant strain, *shrinkage and swelling*, the time dependent deformations caused by changes in specimen moisture content and *thermal dilatations*. Only creep is discussed in this document because approximate formulas relating creep and relaxation are

available (Appendix D), and because the magnitude of the expected shrinkage, swelling and thermal dilatations of most materials is considered to be well known. However, the interaction between moisture and temperature changes and creep is discussed. A further reason for concentrating on creep is that it is the most studied rheological property.

A phenomenological description of creep in a uniaxial state of stress is given. The underlying processes responsible for creep are only occasionally mentioned. They must be studied on a fundamental physical and chemical level.

Notation

The analysis of deformational behaviour is facilitated by a separation of the total deformation (strain) into the following components (see FIG. 1): instantaneous elastic, delayed elastic and viscous strain. A fourth component, the plastic, or instantaneous permanent strain turns out to be without importance at the stress levels considered in this work. It is therefore not included in FIG. 1. The curve OA is called the *creep curve*.

The first loading of a material produces greater deformation than subsequent unloading or reloading. The first creep curve is called the *virgin creep curve*.

The magnitude of creep naturally depends on the kind and quality of the material. Other influences are: time, stress, temperature, equilibrium moisture content and variations of the latter parameters. The qualitative influences of these factors on creep are shown in FIG. 2. Each will be commented on below.

Key words:

rheology, building materials, long-term deformations (moisture, temperature), creep, modulus of elasticity

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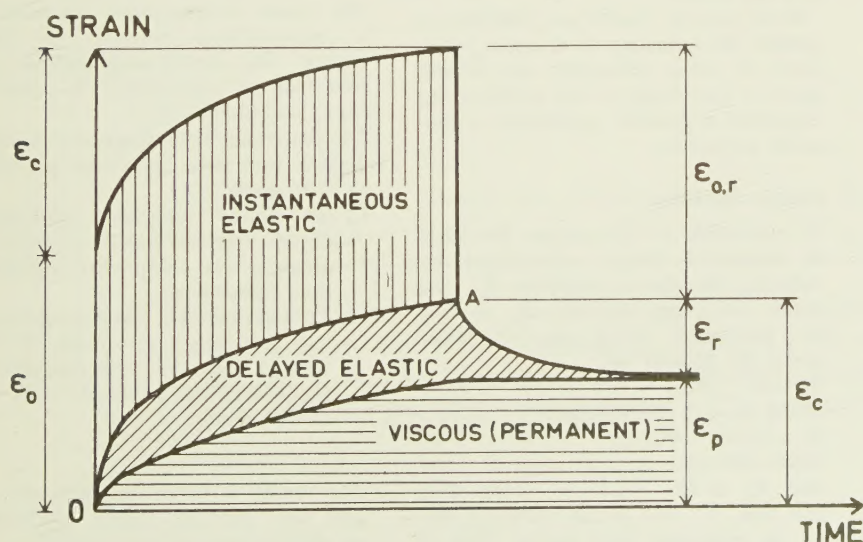


FIG. 1 Strain components during loading and unloading. ϵ_0 is initial strain, ϵ_c creep strain, $\epsilon_{o,r}$ initial recovery, ϵ_r instantaneous time dependent recovery, and ϵ_p is permanent strain.

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Time

Many mathematical expressions have been proposed to describe the time dependence of virgin creep ϵ_c (FIG. 2 A). The most universally applicable is the power function

$$\epsilon_c(t) = A \cdot t^b \quad (1)$$

A and b are constants; $b < 1$.

The power function is linear in a log-log plot with a slope of b and an intercept of $\log A$ at $t = 1$. The power function describes the virgin creep curve of most building materials in climatic equilibrium, for compression, tension and flexural tests. It also provides an approximate description of creep of materials exposed to periodically fluctuating climates, for example outdoor structures. The constant b appears to be independent of stress, temperature and moisture condition. The influences of these factors are reflected by changes in the value of A . (Note that b depends on the method used to determine the instantaneous deformation).

Stress

The influence of stress, σ , on the virgin creep (see FIG. 2 B) is most accurately expressed by the function:

$$\epsilon_c = K_1 \cdot \sinh(K_2 \sigma) \quad (2)$$

K_1 and K_2 are constants. When fitted to experimental data, this function can usually to a good approximation be considered to be linear up to 30 % of the fracture stress.

The stress has a different influence on the various strain components. It appears that the instantaneous and the delayed elastic strains are linearly related to stress up to very high stress levels. It is postulated as a working hypothesis that the final delayed elastic strain always amounts to a certain fraction of the instantaneous strain for a given type of material in equilibrium at a certain climate. This postulate implies that the nonlinear relationship between creep and stress, eq. (2), is caused by the nonlinearity of the viscous component. More research is needed to establish this postulate.

Boltzmann's superposition principle states that the total creep produced by several load-unload increments can be computed as the algebraic sum of the creep caused by each increment. This principle is discussed, and it is pointed out that it may be applied to building materials provided that the delayed elastic and viscous components are separated, and the principle is applied to each independently.

Climate

Temperature and moisture influence creep in two ways. Firstly, creep in climatic equilibrium is greater at higher temperatures and relative air humidities than at lower (see FIG. 2 C and D). Secondly, creep increases if the climatic equilibrium is disturbed while the load is on (see FIG. 2 E). An illustration is given in FIG. 3 which shows creep of wooden beams

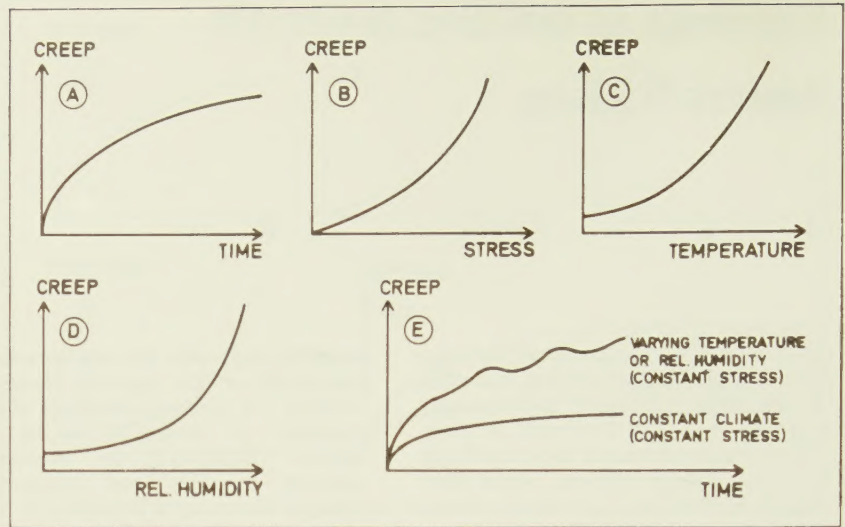


FIG. 2 Sketch showing the influences on creep of: A Time. B Stress. C Constant temperature. D Constant relative humidity (constant moisture content). E Time given constant climate (temperature and relative humidity) or varying climate.

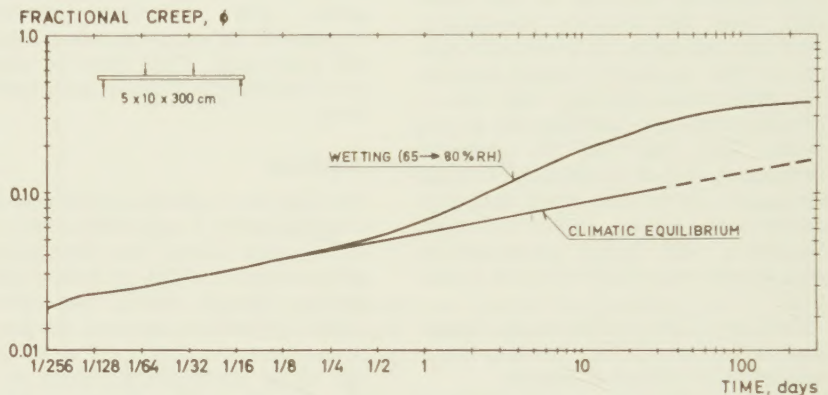


FIG. 3 Fractional creep vs. time for pine beams in climatic equilibrium and during wetting.

in equilibrium at 65 % relative humidity (RH), and while wetted from equilibrium in 65 % RH to equilibrium at 80 % RH. The rate of creep increases during wetting and decreases when wetting is completed after about 30 days, thus producing the S-shaped curve. Pickett proposed that the internal stresses arising during climatic changes cause the observed increase in creep. His hypothesis is discussed in some detail.

Some rules of thumb are available to predict the influence of climatic variations on creep. However, our present state of knowledge is not sufficient to formulate a general quantitative calculation procedure.

Creep modulus

It is possible to compensate for creep in structural design calculations by reducing the elastic modulus, E_o . The result is a creep modulus, E_L . It may be expressed using the fractional creep, Φ , defined by:

$$\Phi(t) = \epsilon_c(t)/\epsilon_o \quad (3)$$

Using eq. (1) one obtains:

$$E_L = E_o/(1 + \Phi_y \cdot t^b) \quad (4)$$

where the time, t , is in units of years and Φ_y is the fractional creep after one year. Φ_y and b can relatively easily be estimated from log-log plots of

virgin creep curves. Some numerical values of E_o , Φ_y and b for different materials are given in the report. Type of loading, temperature and moisture condition must be specified. The reported values vary as follows: Φ_y between 0.2 and 12.0, b between 0.1 and 0.4 but with a single value of 0.7.

Recommended future research

The report demonstrates the need for a standardized method of creep testing. This would eliminate the present lack of consistency in determining ϵ_o and thereby E_o .

The recommended research work can be divided into two categories according to their goals:

- ☐ To obtain empirical information about the influence of climatic changes on creep and superposition calculations.
- ☐ To elucidate the fundamental chemical and physical causes of creep with a view to improvements in manufacturing processes and use of materials.

The report is based on literature studies and personal creep experiments on cellular concrete and wood.

RHEOLOGY OF BUILDING MATERIALS

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PREFACE

The use of the word rheology has become more and more common in civil engineering in recent years, reflecting a growing awareness of the universality of rheological phenomena. Creep and shrinkage are two such phenomena well known to the civil engineer. I have attempted to survey the creep behaviour of building materials using rheological methodology. The work is based on literature studies and on experimental studies which I have performed or been associated with in a supervisory capacity.

It is difficult to cover such a wide field in a limited space. I am aware that valid objections can be made to my presentation. However, I hope that my work will be useful as a textbook and as a guide for practicing engineers and that it will provide inspiration for research workers.

I want to express my warm appreciation of the encouragement and inspiration given by Professor S.G. Bergström. The technical assistance and helpful discussions provided by many of my co-workers are gratefully acknowledged. Especially I want to thank E.J. Sellevold, Ph.D., for his close scrutiny of the English manuscript, Mrs. Hanne Schmidt for her enthusiastic typing and Mrs. G. Wrangel and Mrs. B. Rosenø for their excellent draftmanship

Most of the work was performed during my employment in the Division of Building Materials, Lund Institute of Technology, Sweden, 1964-1971. It was, however, completed in connection with my present post at the Civil Engineering Department of Danmarks Ingeniørakademi.

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Copenhagen, April 1972,

Anders Nielsen

CONTENTS

NOTATIONS	6
1. INTRODUCTION AND SCOPE	9
2. GENERAL REMARKS	13
2.1 History	13
2.2 Definitions of rheology	15
2.3 Rheology of building materials	16
2.4 Terms	19
3. FACTORS INFLUENCING LONG-TERM DEFORMATIONS	25
3.1 Survey of factors	25
3.2 The scatter of deformations observed in practice	28
4. PARAMETERS DESCRIBING LONG-TERM DEFORMATIONS	33
4.1 Parameters representing total deformations	33
4.2 Parameters based on creep deformations	39
4.3 Stress parameters	44
4.4 Error propagation	44
5. TIME DEPENDENCE	49
5.1 Separation of initial deformation and creep	49
5.2 Time functions in general	57
5.3 The power function	61
5.4 The simple rheological models	69
5.5 The generalized rheological models	72
6. STRESS DEPENDENCE	75
6.1 Stress and virgin creep	75
6.2 Strain components	83
6.3 The Boltzmann superposition principle	97
6.4 Graphical superposition	105
6.5 Mathematical superposition	109
7. INFLUENCE OF CLIMATE	114
7.1 Constant climate in general	114
7.2 Creep at constant temperature	115
7.3 Creep under constant moisture conditions	121
7.4 Creep under varying climatic conditions	125
7.5 Creep under unidirectional transport	132
7.6 Desorption creep of concrete	141
7.7 Cyclical variations of climate	144
7.8 Variations of climate in practice	147
8. CONCLUSIONS	154
8.1 General descriptions of creep behaviour	154
8.2 Recommended future research	157

APPENDIX A.

AN EXPERIMENTAL STUDY OF BASIC CREEP OF WOOD	159
A.1 Summary of the experiments	159
A.2 Results	161
A.3 Conclusions and discussion	161

APPENDIX B.

AN EXPERIMENTAL STUDY OF SORPTION CREEP OF WOOD	169
B.1 Summary of the experiments	169
B.2 Results	170
B.3 Conclusions concerning species, grade, and stress	171
B.4 Specimen size and climate	171
B.5 Moisture measurement	183
B.6 Main conclusions	184

APPENDIX C.

CREEP OF CELLULAR CONCRETE	188
C.1 Background	188
C.2 Experiments	188
C.3 Results	193

APPENDIX D.

CREEP AND RELAXATION	203
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REFERENCES	205
------------	-----

NOTATION

Letters

A	Constant term in the power function eq. (5.2), μS or μm
B	dimension of specimen, cm or mm
B	a constant
C	a constant
E	a modulus, kp/cm^2
E_0	Young's modulus, kp/cm^2
E_L	creep modulus, kp/cm^2
H	dimension of specimen, cm or mm
J	a compliance, $\mu\text{S} \cdot (\text{kp}/\text{cm}^2)^{-1}$
J_0	initial compliance, $\mu\text{S} \cdot (\text{kp}/\text{cm}^2)^{-1}$
J_L	creep compliance, $\mu\text{S} \cdot (\text{kp}/\text{cm}^2)^{-1}$
ΔJ	specific creep strain, $\mu\text{S} \cdot (\text{kp}/\text{cm}^2)^{-1}$
K_1	a constant
K_2	a constant
K_3	a constant
L	length of specimen, cm or mm
P	force, kp
RH	relative humidity of the air, %
S	external surface of specimen, m^2 or cm^2
T	temperature, $^{\circ}\text{C}$
V	volume of specimen, m^3 or cm^3
V/S	volume-surface ratio, m or cm
a, b, d	constants
b	exponent in the power function t^b
e	base of the natural system of logarithms
$f(t)$	a function of time
$g(\sigma)$	a function of stress
$h(T)$	a function of temperature
$i(u)$	a function of moisture content
k	a constant
n	exponent in the function σ^n
p	level of statistical significance, dimensionless
t	time, in days or years
u	moisture content, percent of dry weight
w/c	water-cement weight ratio, dimensionless
x	a general space coordinate

Greek letters

α	a constant
β	measure for the distribution of retardation times
δ	deflection, mm or μm
δ_0	initial deflection, mm or μm
δ_c	creep deflection, mm or μm
ϵ	strain, deformation per unit length, μS
$\dot{\epsilon}_c$	creep rate (time unit) $^{-1}$
η	viscosity, (time unit) $\cdot \text{kp}/\text{cm}^2$
σ	stress, kp/cm^2
$\dot{\sigma}$	stress rate, kp/cm^2 per time unit
σ_B	ultimate strength, modulus of rupture, kp/cm^2
σ/σ_B	stress intensity, % or dimensionless
τ	retardation time
μS	microstrain, abbreviation for 10^{-6}
Δ	change of something
Φ	fractional creep, % or dimensionless
Φ_h	fractional creep after one hour
Φ_d	fractional creep after one day
Φ_y	fractional creep after one year
$\Psi(t)$	a function of time

Subscripts

c	creep -
e	- after infinite time, final -
h	- after one hour
i,j	summation indices
L	Long-term -
p	permanent -
r	recovery -
s	moisture induced strain, shrinkage or swelling
t	- at a certain time
B	failure-, crushing-, ultimate-
tot	total -
y	- after one year

Statistical symbols

$\bar{y}$ or $\bar{y}$	mean value of y
s or $s\{y\}$	standard deviation of y
c or $c\{y\}$	coefficient of variation of y , $c\{y\} = s\{y\}/\bar{y}$
$v\{y\}$	variance of y
p	level of statistical significance

Conversion factors

$$1 \text{ kp} = 1 \text{ kgf} = 9.81 \text{ N}$$

$$1 \text{ kp/cm}^2 = 9.81 \text{ N/cm}^2 = 0.0981 \text{ N/mm}^2 \sim 0.1 \text{ N/mm}^2$$

1. INTRODUCTION AND SCOPE

Building materials do not reach their final deformation immediately after a load is applied, as is often assumed in design calculations.

The deformation in these materials may continue for years. This may result in problems such as greater deflections than expected, loss of prestress, and loosening of fasteners. On the other hand, long-term deformations have saved many structures with stress concentrations from severe cracking. Furthermore, the importance of long-term deformational behaviour is that the design procedures involving new materials, such as polymers, are in many cases best based upon an expected life-time, i.e. upon a certain deformation by a certain time.

The aim of this thesis is to give a survey of the long-term deformational behaviour, the rheology, of building materials. The survey will show to what extent general rules may be established for the materials under consideration. Furthermore, I will consider the variations of the parameters used in structural design, and I will attempt to distinguish between the rheological methods used in structural design and those used in materials science.

This thesis, therefore, is written for teachers of pure and applied mechanics, materials science, and materials technology. It is my hope that research workers in these fields will get some inspiration from my viewpoints also, and that designers will find some numerical values.

In contrast to the conventional method of discussing several properties of a given material, I will consider how a given property manifests itself in several materials. This approach is rather useful for obtaining general rules. For example, the general applicability of the power function to describe time dependence is shown, and it is demonstrated that

TABLE 1.1. BUILDING MATERIALS

GROUP	MATERIAL	HYGRO-SCOPICITY
Silicates	Concrete	+
	Hydrated calcium-silicates (cellular concrete, sand-limestone)	+
	Ceramics (tile)	(+)
	Stone	(+)
Organic materials	Wood	+
	Wood-based products (particle board, fiber board, plywood)	+
	Polymers	+ -
Metals	Steel	-
	Al-alloys	-

climatic variations in principle have the same macroscopic effect on creep of all types of hygroscopic materials.

Building materials are simply the solids which are used in buildings. The present work is based mainly on studies of concrete, cellular concrete, wood and woodbased products, i.e. highly hygroscopic materials. Polymers and metals have also been studied, but to a lesser extent. The behaviour of prestressing steel and aluminium alloys has been treated briefly in the previous edition of this book (*Nielsen (1968a)*).

Further limitations are:

- * Only deformations under nominally uniaxial stress (tension, compression, and bending) are considered (the behaviour in multiaxial states of stress is yet so little known that few general conclusions can be drawn);
- * only static conditions are considered, mainly at stress levels normally used in design;
- * vibrational properties are not considered;
- * only temperatures normally encountered at building sites are considered;
- * only creep is considered. (Pure shrinkage is wellknown for most materials.)

The interaction between creep and shrinkage is discussed.

The listing of the hygroscopicity of the materials in TABLE 1.1 indicates where this interaction must be taken into account.

Rheology deals mainly with creep and stress relaxation. Creep is the timedependent deformation of a material under constant load, and stress relaxation is the timedependent decrease of stress under a constant strain. These two phenomena are "kissing cousins" in the sense that they both are manifestations of the same microscopic processes. The theory

of viscoelasticity relates the two phenomena mathematically (see Appendix D). Here it is mainly creep which is considered. This is partly in order to be brief, partly because the most commonly used experiments are creep experiments, and partly because mathematical interrelations between creep and relaxation exist.

The main emphasis is on a search for general rules to describe creep phenomena. I have not tried to discuss the causes of the deformations (only tentative explanations are occasionally given) since these causes have to be treated on a fundamental physical and chemical level.

The variations in long-term deformations are great and the influences on their development many. It is no wonder that the designer is often forced to guess about the influences of creep on his calculations. I hope that this work may provide a foundation upon which he can base "educated guesses".

2. GENERAL REMARKS

2.1 History

Until the beginning of this century all building materials were taken from nature and used either directly, as for example wood and stone, or after undergoing a manufacturing process developed over centuries, as for example bricks and mortar. The properties of the materials and the rules for their use were therefore relatively well-known.

From the beginning of this century the modern chemical industry started to assert itself in the field of materials with products such as rubber, plastic, paint, oil and cellulose products. The deformational behaviour of these materials could not be described sufficiently well by the classical theories of elasticity and plasticity. Therefore, in the last part of the 1920's, American chemists under E.C. Bingham developed methods to describe the behaviour of these substances. They called this branch of science *rheology* after the Greek words $\rho\epsilon\omega$, to flow, and $\lambda\omicron\gamma\omicron\varsigma$, science.

Parallel to this development there was a recognition within mechanical and civil engineering of the necessity of considering the long-term deformations of structural materials. The methods developed in these fields for the description of such long-term deformations could be used in chemistry and vice versa, and so the field of rheology became interdisciplinary.

Today rheology is a "cross-science" based upon mathematics, physics and chemistry, and applied to solid mechanics, structural design, geology, soil mechanics, food production, the polymer industry, the paint industry, cellulose and steel production and medicine.

The tool which characterizes classical rheology is the concept of a rheological model. This concept is

based upon the assumption that the deformations of all materials may be classified as either elastic, viscous or plastic. From this assumption mathematical expressions for the deformational behaviour can be derived. These expressions correspond to the deformations of a mechanical system consisting of springs (elasticity), dashpots (viscosity) and friction elements (plasticity), a so-called rheological model. The method is described by *Reiner (1960)* or *Scott-Blair (1969)*, among others, and it is reviewed here in section 5.4. This method is not the only way of describing deformations and it has been much criticized in works by *Barkas (1949)*, *Ruetz (1966)*, *Kaumann (1966)* and others. One may agree with the critical remarks, but it must be admitted that it was this concept of a universal theory of deformation which formed the basis for rheology as a "cross-science".

Scott-Blair (1969) has a detailed description of the history of rheology. He gives quotations on rheology from the classical Greek, Roman, Arabian, Indian and the medieval European literature.

The idea of treating the deformational behaviour of all kinds of materials within the framework of a single science means that research workers from quite different branches of science and industry become aware of the fact that they have problems which are alike, and that they can get together to solve them. This was particularly true for the phenomenological description of deformations, which initially was the main subject of rheology.

The other aspect of rheology concerns the causes of deformations. Also in this type of work scientists can be inspired to solve their own problems by looking at solutions in other groups of materials. *Forslind (1966)* points out that rheology in the more advanced branches of science has changed from being primarily descriptive to becoming mainly explanatory.

2.2 Definitions of rheology

Rheology means, as mentioned above, *the science of flow*. The historical explanation for this is that the description of the flow behaviour of the new synthetic materials was the first problem for the rheologists.

According to *Scott Blair (1969)*, the original version of the definition of rheology as stated at the foundation of the American Society of Rheology in 1929 was: "Rheology is the study of the deformation and flow of matter". Since flow is a type of deformation, this definition is tautologous. This was pointed out by Reiner, and in a recent list of rheological terms (*Eirich (1967)*), the definition was changed to "... the study of the deformations of materials, including flow."

As more and more branches of science begin to make use of rheology, this short definition has become inadequate. If on the other hand one wants a more exact definition of rheology, covering all applications, it becomes too detailed. In either case, it is of little practical importance. So, in the literature, one finds that research workers who define rheology give their own formulation to suit their own branch of science:

Reiner, one of the founders of classical rheology, follows the original definitions quoted above but also includes the classical theories of elasticity, plasticity and hydraulics in rheology (*Reiner (1960)*).

F.H.G. Odquist (professor in mechanics) on the other hand considers it most practical to see rheology as that part of the description of deformations which is not included in the classical theories (*Odquist (1965)*).

E. Poulsen (professor in pure and applied mechanics) gives two definitions, one regarding materials and one regarding structures. Concerning materials:

Rheology of materials is the science of the time-dependent physical conditions including flow- and fracture-conditions (Danish: "Materialereologi er læren om fysiske betingelser samt brud- og flydebetingelser, når disse er tidsafhængige"). Concerning structures: Rheology of structures is the science of strength and deformations with regard to the influence of time (Danish: "Konstruktionsreologi er styrke- og deformationsslære under hensyntagen til tidens indflydelse", *Poulsen (1967)*).

E. Forslind (physicist) defines the rheology of today as that branch of science which deals with the irreversible deformational properties of matter (Swedish: "Med reologi menar man i dag läran om materiens irreversibla deformationsegenskaper", *Forslind (1966)*).

K.E.C. Nielsen (concrete research worker) says that rheology is that branch of science which seeks to describe the time-dependent deformations of materials when they are exposed to mechanical actions, as a function of their composition and structure. (Danish: "... den videnskabsgren, som søger at nå frem til en videnskabelig korrekt beskrivelse af materialers formforandringer i tiden, når de udsættes for mekaniske påvirkninger, ... som funktion af materialernes fysisk-kemiske bestanddele og strukturelle opbygning." *K.E.C. Nielsen (1965)*).

It should also be pointed out that the word "rheology" is not used by *Nadai (1963)*, although a great deal of his book deals with viscosity and plasticity.

2.3 Rheology of building materials

After having studied the deformational behaviour of building materials I propose the following definition as suitable to this branch of science:

Rheology of building materials deals with the description and the explanation of the time dependent deformations caused by mechanical load and changes in moisture content and temperature.

In this definition both description and explanation are mentioned. The description of the deformations gives the equations, parameters, and coefficients necessary for structural design. This part of rheology is the most important from the practical point of view. The explanation of the deformational behaviour of materials on a fundamental level is a prerequisite to a systematic improvement in their properties. To give such explanations, basic research based on chemistry, physics and mathematics must be used.

For the solid materials, the rheological properties to be studied are *creep*, the deformation under constant load, *relaxation*, the reduction of stress under a constant strain, and *vibrational behaviour*, the behaviour during repeated loading. For the hygroscopic materials we have the further problems of *shrinkage* and *swelling*, i.e., the deformations during moisture exchange. Some research workers want to exclude shrinkage and swelling from rheology. This exclusion is based upon an assumption of creep and shrinkage as two independent phenomena. This assumption is a convenient approximation when the goal is to improve the practical calculations of deformations, but it is only an approximation, as shown by recent research on wood and concrete (*Barkas (1949), Ruetz (1966), Wittmann (1966)*).

The deformational behaviour of building materials during the manufacturing process, where all except wood and stone are in the fluid state, is not treated in the rheology of building materials. An exception is the manufacturing of concrete, which traditionally is dealt with in building materials science, since

TABLE 2.1. COMPONENTS OF DEFORMATION

	Reversible (Recoverable)	Irreversible (Permanent)
Instantaneous (Initial)	Instantaneous elastic	Plastic
Time dependent	Delayed elastic	Viscous

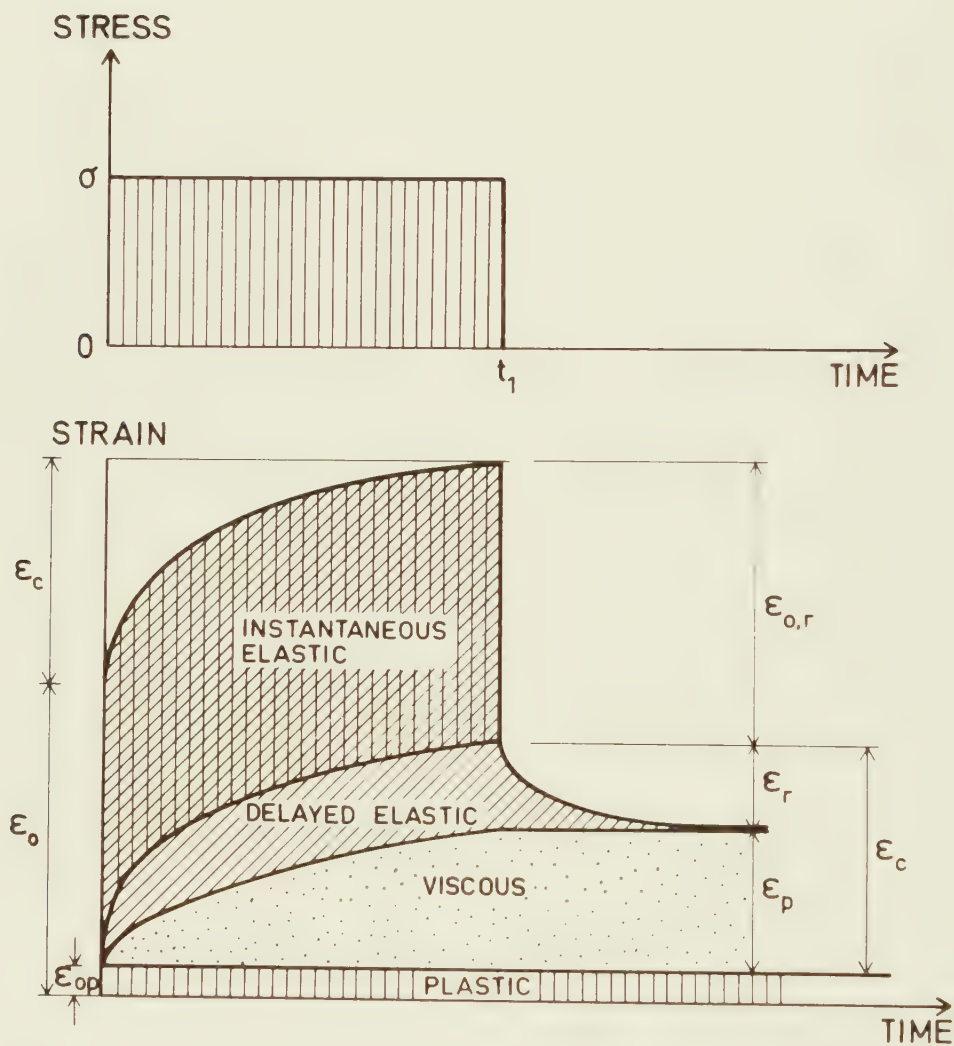


FIG. 2.1 Strain vs. time for a specimen of a hypothetical material. The stress is constant, σ , until the time t_1 , when the specimen is unloaded. Strain components are shown.

the builder is often also the manufacturer. Fresh concrete was briefly dealt with in the first edition of this book (*Nielsen (1968a)*).

2.4 Terms

This section deals with the terms which are used to describe the different components of deformation for a solid, as well as with terms used to describe creep phenomena observed under certain special circumstances.

The deformation vs. time diagram for a hypothetical material in a uniaxial test is shown in FIG. 2.1. There is great uncertainty in the literature concerning the proper terms to use for the different components of the deformation. One finds words such as plastic, viscous, delayed elastic, elasto-plastic and viscoelastic, but without any consistent definitions. I have tried to use the terms which are shown in FIG. 2.1 and in TABLE 2.1 (partly in accordance with *Hansen (1960)*). The components are characterized by their time dependence and reversibility (or recoverability) (FIG. 2.1). The words elastic, plastic and viscous are connected with time dependence and reversibility as shown in TABLE 2.1. These words originate from the proposition of classical rheology that deformations in all materials can be assigned to either elasticity, plasticity or viscosity.

Immediately upon application of a load the material gets an *instantaneous* or *initial* deformation. Instantaneous and initial are used interchangeably here. The instantaneous deformation can be divided into a reversible part, the *elastic*, and an irreversible part, the *plastic*, deformation. The magnitude of the plastic deformation can be determined only through immediate unloading. The meaning of the term instantaneous is discussed in Section 5.1.

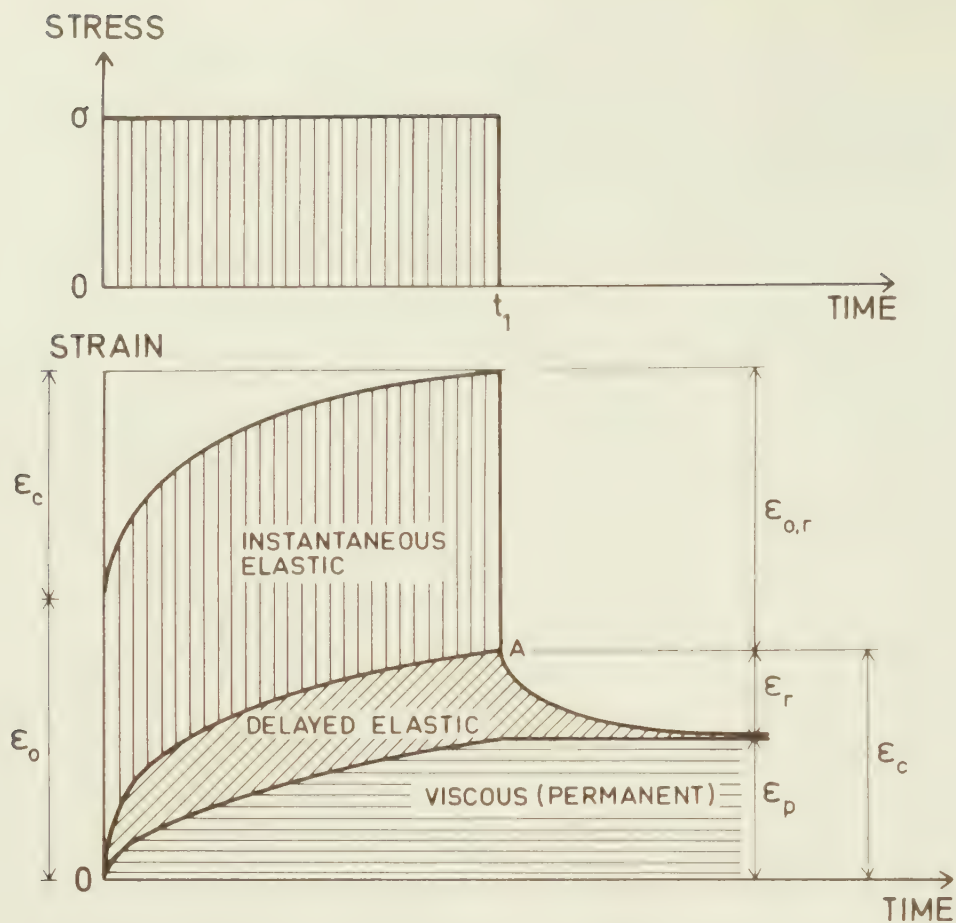


FIG. 2.2 The same diagram as in FIG. 2.1, but without the plastic strain. The curve 0A is the "creep curve".

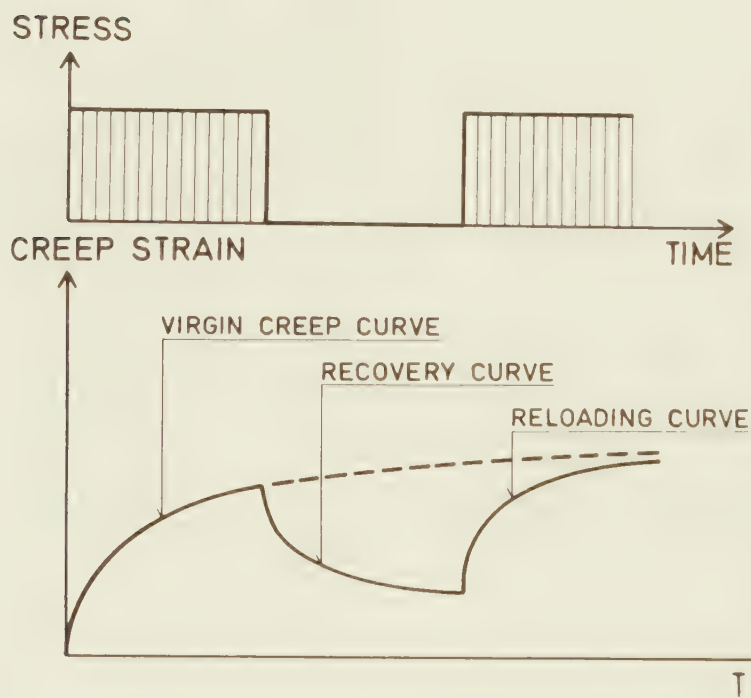


FIG. 2.3 Terms used in connection with the creep behaviour under first loading, unloading and second loading.

After zero time the material creeps. The term *creep* (Sw: krypning) is used about the total time dependent deformation. After unloading it is seen that the time dependent deformation consists of a *reversible* (recoverable) part, the *delayed elastic* deformation, also called the *recovery* (see below), and an *irreversible* (irrecoverable) part, the *viscous* deformation. The viscous and the plastic components are also often referred to as the *permanent* deformation.

The term *recovery* (Sw: återhämtning) is used here for that part of the creep which is recoverable. *Neville (1960a)* uses the term *creep recovery*. (Note that in metallurgy the term *recovery* is used to describe the process of internal stress relief at elevated temperatures.)

Many building materials do not show any measurable plastic strain at low stresses. In this case the strain-time diagram looks as shown in FIG. 2.2. When creep of building materials is discussed and reported, the instantaneous elastic strain in the diagrams is usually omitted, i.e., only the *creep curve* OA is shown.

In connection with repeated loading and unloading it is practical to use the term *virgin creep* about the creep during the first loading period. The virgin creep is usually larger than both the recovery and the creep during the *reloading*, see FIG. 2.3.

When a hygroscopic material is dried or wetted when under load, the creep is greater than when no moisture transport happens, see FIG. 3.1. The term *basic creep* is used about creep of a specimen which is not wetted or dried from the environment during loading, whereas the term *sorption creep* is used here for creep with simultaneous sorption.

Hansen (1960) and *Neville (1970a)*, among others, use the term *drying creep* in this connection. However, this term is especially relevant to concrete drying

from the wet state. The term sorption creep indicates the influence on creep from drying (desorption) as well as from wetting (adsorption).

As will be seen later, creep during changing temperature is also higher than the basic creep. Here one could introduce the term *thermal creep*. Creep during changing moisture content and temperature, as is the case under practical conditions, could be called *climatic creep*. However, I will introduce no new terms, but rather explain as the need arises.

As mentioned above, hygroscopic materials also experience the time- and moisture-dependent deformations *shrinkage* (Sw: krympning, Da: svind) and *swelling*. In creep studies one normally attempts to exclude these effects by measuring pure shrinkage on a dummy specimen. Creep is then reported as the difference between the actual creep curve and the shrinkage curve, see FIG. 7.10. This procedure is only an approximation, since the two phenomena are not independent, as already pointed out. However, I have followed this common practice in my treatment, since most data are reported this way.

Tough materials (metals, wood, plastics) will at different stress levels or temperatures exhibit strain vs. time curves such as those shown in FIG. 2.4. The upper curve may be split into three regions: one where the rate of deformation is decreasing, one where it is constant, and one where it is increasing. The creep in these regions is referred to respectively as *primary*, *secondary* and *tertiary*. The tertiary creep ends in rupture.

For brittle materials (concrete, cellular concrete) this division has little practical significance. Secondary and tertiary creep have been reported only by a few authors, for example by Wittmann (1971). The creep of building materials observed under practical conditions is most often primary, and it is this type of creep which is the main topic of my work.

The word *flow* (Sw: flytning) is used in the literature interchangeably with the words plastic, delayed elastic and viscous. I have preferred not to use the word flow because of this confusion. - The term *viscoelastic* is taken by many authors to include elastic, delayed elastic and viscous deformations.

After this discussion of terms it should be pointed out that it is more important to describe a certain behaviour accurately than to associate it with a certain terminology.

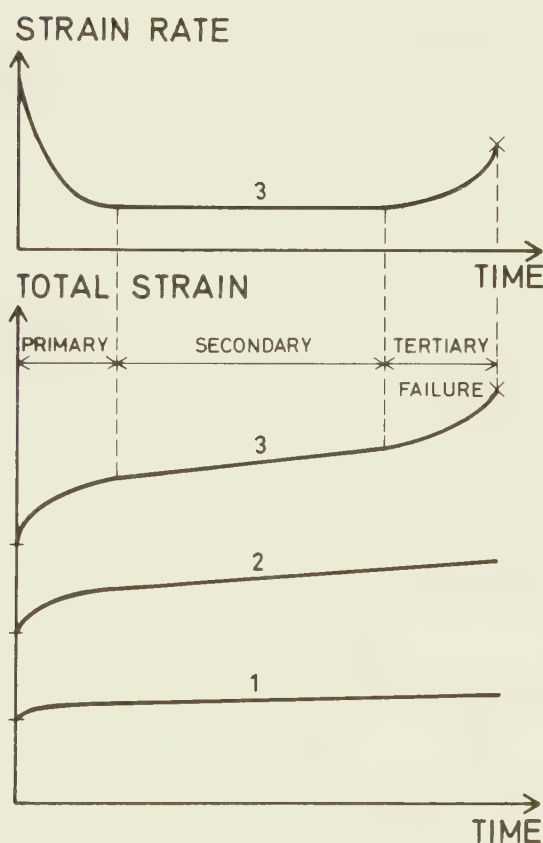


FIG. 2.4 Bottom: Possible strain vs. time curves at different constant stresses $\sigma_1 < \sigma_2 < \sigma_3$ or at different constant temperatures $T_1 < T_2 < T_3$. Top: Strain rate during primary, secondary and tertiary creep.

TABLE 3.1. FACTORS INFLUENCING TOTAL LONG-TERM DEFORMATIONS

Factor	Remark
Group of material (concrete, wood, steel ...) Type and quality of material within the group	Internal structure-dependent factors
Time Stress level and history Temperature level and history Size of specimen	External factors for as well hygroscopic as unhygroscopic materials
Moisture level and history	External factor for hygroscopic materials

In this chapter a survey is given of the internal and external factors influencing the long-term deformations. In the following chapters the influences of time, stress level, loading history, temperature and moisture will be treated separately. TABLE 3.1 gives a tabulation of the factors.

3.1 Survey of factors

The internal structure of a material, i.e. the configuration of atoms or molecules, influences the creep behaviour. Thus the group to which a material belongs (whether it is wood, steel, concrete, polymers etc.) determines its rheological behaviour.

Furthermore, the microstructure varies within a given group and has a pronounced influence on creep, i.e. the type and quality of a particular material within the group also influence the creep. The factors determining the microstructure are numerous. In concrete, for example, the parameters are: type and fineness of cement, additives, aggregate type and fineness, water-cement ratio, aggregate content, mixing time, vibration, curing, and age at loading.

The external factors are time, stress, temperature and humidity conditions, and FIG. 3.1 indicates the influences of these. Specimen size can be seen as an external factor, too.

For a given material, hygroscopic as well as unhygroscopic, at a given type of constant loading (bending, tension, compression or shear), the magnitude of creep increases with time.

Stress influences the creep development both in the way that higher stress produces more creep, and in the way that the material is mechanically conditioned, so that upon a second loading the creep tendency is decreased (see FIG. 2.3). Hence the creep behaviour of a material at a certain moment depends on the

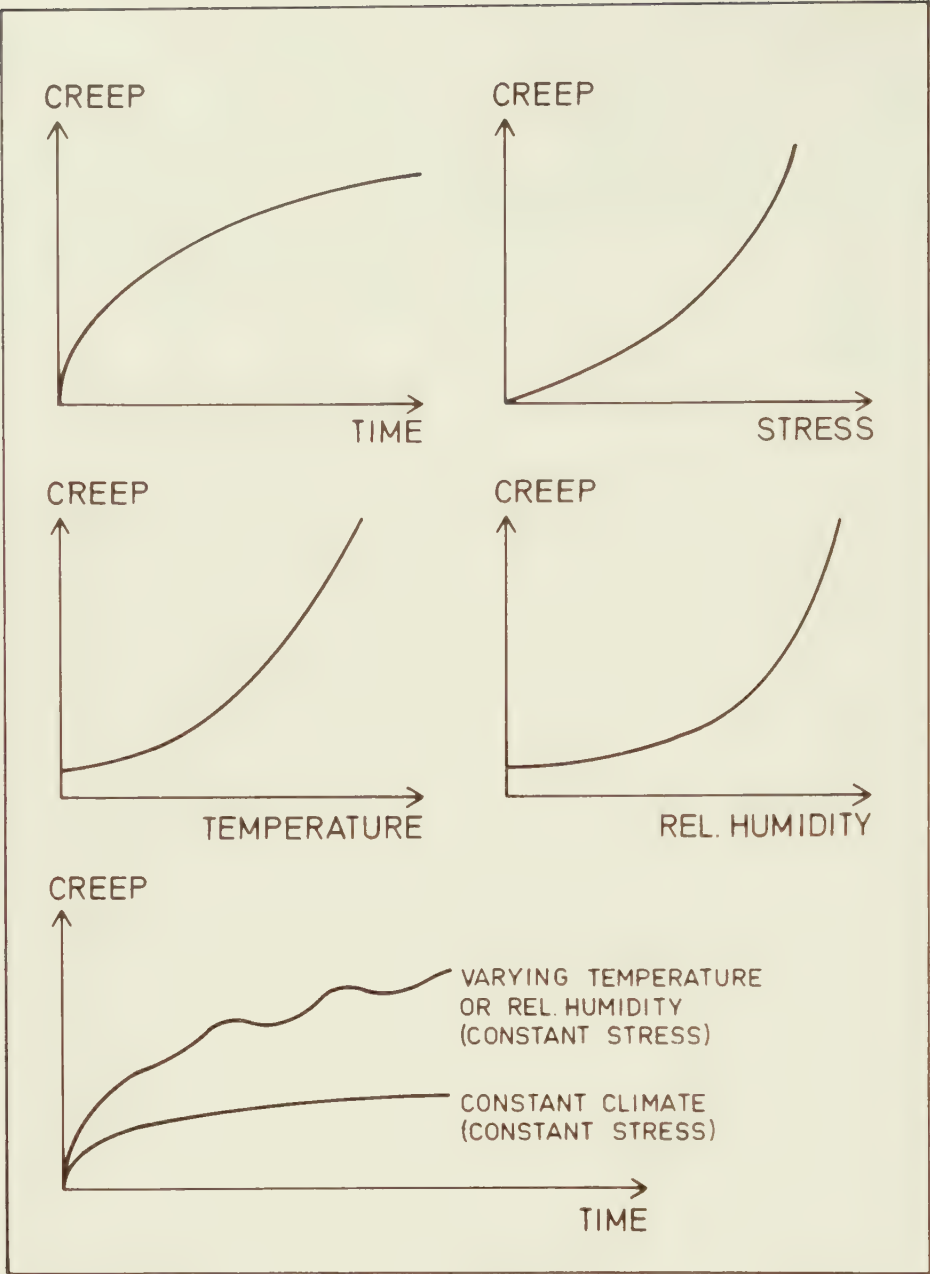


FIG. 3.1 Graphs indicating, in principle, the influences of different factors on virgin creep.

previous stress history.

Creep becomes more pronounced at higher levels of temperatures. Furthermore, changes in temperature during a creep experiment will give rise to internal stresses in a specimen and thereby influence the creep. The internal stresses depend on the gradient of temperature with respect to time as well as to specimen dimensions.

Specimen dimensions also become important when the material contains faults, for instance knots in wood, in the way that a large specimen is more likely to contain faults, and therefore deforms more easily. *Curtu et al. (1969)* have treated this phenomenon for wood. A parallel to the Weibull theory of the influence of dimensions on strength can be drawn. However, the main effect of specimen dimensions on creep is the influence they exert on the various gradients.

For the hygroscopic materials the problem of moisture dependence enters. Moisture also influences creep in two ways. Firstly, a specimen in equilibrium at a high relative humidity in the surrounding air creeps more than a specimen in equilibrium at a lower RH. Secondly, if a change in the moisture content occurs during the experiment, it will also cause a change, most often an increase, in the magnitude of creep. These phenomena are illustrated on FIG. 3.1.

The influence on the creep strain, ϵ_c , of the above mentioned factors can be expressed as follows:

$$\epsilon_c = F[t, \sigma(t, x), T(t, x), u(t, x), V] \quad (3.1)$$

where t is time,

σ is stress,

T is temperature,

u is moisture content,

x is a general space coordinate, and

V is volume of specimen.

The number of variables in this expression indicates that an exact mathematical formulation of creep will be very complicated, even if the interdependence of the variables is known.

Most creep experiments in the laboratory are made under well-defined climatic conditions. Most often humidity and temperature are held constant and it is supposed that the influencing factors are independent of each other. In this case the equation (3.1) can be simplified to:

$$\epsilon_c = f(t) \cdot g(\sigma) \cdot h(T) \cdot i(u) \quad (3.2)$$

Furthermore, often only the influences of time and stress are studied. Then $h(T)$ and $i(u)$ are reduced to constants.

Such simplifications make the experiments and their evaluation easier, but they have also had the effect that the problems concerning basic creep are relatively much more studied than the problems concerning sorption creep, which is of greater practical importance.

3.2 The scatter of deformations observed in practice

The total deformations occurring in practice are the principal concern in building materials rheology. Most often the mean values are treated, but the scatter of the total deformations may not be ignored.

Knowledge of scatter is essential for any rational discussion of the accuracy of design procedures. Furthermore, such knowledge is useful to have when research work is being planned; the possible importance of the factors to be studied can be judged in relation to this.

This problem of scatter will be treated below, and some examples of observed scatter shall be given.

The factors tabulated in TABLE 3.1 influence the mean

value as well as the scatter of the deformations. When some presumably identical specimens are subjected to the same external influences, the mean value is determined by the internal as well as the external factors. The scatter is determined mainly by the internal factors which determines the microstructure. But the interactions between internal and external factors may also influence the scatter. These interactions can usually not be separated from the effect of the internal factors. The interactions are mentioned here to indicate a possible contributing reason for the great scatter in long-term deformations which is seen in practice.

The first example deals with the total deflections of simply supported beams of timber and gluelaminated wood (*Nielsen, Eriksson, Gustavsson (1970)*). These deflections were measured as part of an evaluation of the creep deflection of timber in existing buildings. The specimen types and the legend are shown in TABLE 3.2. FIG. 3.2 shows the measured total deflections of all the beams plotted vs. year of erection. It is seen that the ranges of the different sets of measurements are rather great, in some cases greater than the mean value. The materials quality is shown by the fact that the gluelaminated beams (dark symbols) have much less scatter than the beams of normal timber.

The second example is taken from my investigation of cellular concrete described in Appendix C. The specimens in this investigation formed a random sample of materials as supplied in practice. FIG. C.7 shows the creep compliance vs. time for the specimens kept at 43% RH while under load. The double residual standard deviation for the specimen set is plotted on both sides of the mean. This interval (approximately the 95% confidence interval) can be taken as a measure of the scatter to be expected in a delivery of cellular concrete. It is seen to be rather big

TABLE 3.2. INVESTIGATION ON DEFLECTION OF TIMBER BEAMS IN PRACTICE

Specimen type	Year of erection	Wood species	Climate	Number of measured specimens	Symbol in FIG. 3.2	Length of beam, m
Timbers in floor	1850	Oak	in	6	☒	3.8
Timbers in floor	1870	Pine	in	2	Δ	3.4
Timbers in floor	1870	Pine	ih	6	∇	{ 6.0 7.9
Timbers in roof	1931	Pine	os	4	0	6.6
Timbers in roof	1942	Pine	os	6	X	4.3
Timbers in roof	1959	Pine	os	6	□	4.3
Timbers in roof	1968	Pine	os	8	⊗	6.7
Glue-laminated beams, floor	1920	Pine	ih	6	▼	4.9
Glue-laminated beams, roof	1964	Spruce	ih	6	▲	13.0
Glue-laminated beams, roof	1968	Spruce	os	2	●	16.0
Glue-laminated beams, roof	1969	Spruce	os	5	■	5.0
Abbreviations for climates:						
			in = inside, no heating			
			ih = inside, heating			
			os = outside, sheltered			

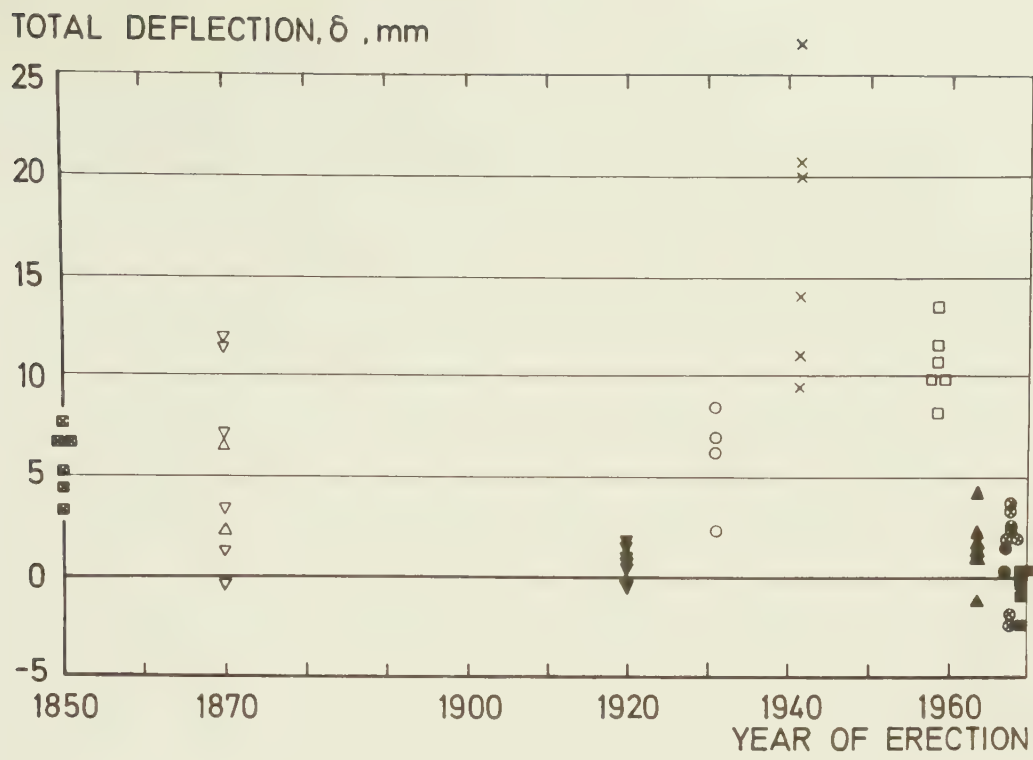


FIG. 3.2 Examples of the scatter in the total de-
formations observed in practice. Investi-
gation of deflections of timber in practice
(Nielsen, Eriksson, Gustavsson (1970)).
Symbols see TABLE and below.

Specimen type	Symbol
Timbers in floor	⊠
Timbers in floor	Δ
Timbers in floor	▽
Timbers in roof	○
Timbers in roof	X
Timbers in roof	□
Timbers in roof	⊗
Glue-laminated beams, floor	▼
Glue-laminated beams, roof	▲
Glue-laminated beams, roof	●
Glue-laminated beams, roof	■

relative to the mean value. (The coefficient of variation at 1 day is $c\{J\} = 11\%$). A peculiarity is that the width of this interval is almost independent of time. One might expect a broader interval at longer times.

As a third example is taken the investigations of creep of wood described in Appendices A and B. In this case also the specimens formed a random sample as supplied in practice. The total deformations were not treated statistically. However, a consideration of the results gives the same impression as the above mentioned investigation of cellular concrete, that the variation of the total deformations within a supply of building materials is rather large.

Recently *Neville (1970b)* quoted some reports on analyses of total deformations of concrete structures. *Keijer (1969)* treats the long-term deflection of four concrete bridges (see Section 5.2). It is my opinion that more investigations ought to be made to enable one to obtain better estimates of the orders of magnitude of mean values and scatter of the total deformations of building materials in practice.

4. PARAMETERS DESCRIBING LONG-TERM DEFORMATIONS

This chapter deals with the parameters used to describe long-term deformations. These parameters can be divided into three groups, namely those representing the total deformation, the creep deformation, and the stress necessary to obtain a certain deformation. The parameters are listed in TABLE 4.1, and the letter code is given in FIG. 2.1. The parameters used in connection with the linear viscoelastic analysis (rheological models) are discussed in Chapter 5.

4.1 Parameters representing total deformations

Parameters representing total deformations can be either the *modulus*

$$E = \sigma / \epsilon_{\text{tot}} \quad (4.1)$$

or the *compliance*

$$J = \epsilon_{\text{tot}} / \sigma \quad (4.2)$$

It follows from this that

$$E = 1/J \quad (4.3)$$

For conventional short-term experiments, where $\epsilon_{\text{tot}} = \epsilon_0$, the subscript 0 is used. For long-term experiments, where $\epsilon_{\text{tot}} = \epsilon_0 + \epsilon_c$, the subscript L is used.

E_0 is the conventional *modulus of elasticity*, also called *Young's modulus*. (E_0 is dependent on the definition of ϵ_0 , see Section 5.1).

The modulus E_L has many names in English: *Ogorkiewicz (1970)* uses "creep modulus", *Richards (1961)* uses "effective modulus", and *McHenry (1943)* "sustained modulus". I will use "creep modulus".

The compliance, J , is, as seen from eq. (4.2), the strain per unit stress. It is also called the *specific strain* (Sw: specifik töjning) and the symbol

ϵ^1 is also often used for this parameter. Here the letter J is employed in accordance with Anglo-American usage. J_0 is in the present treatment called *the initial compliance*, and following the recommendation of *Leadermann and Schwarzl (1961)*, J_L is called the *creep compliance*.

Both E and J are functions of time: $E_L = E_L(t)$ and $J_L = J_L(t)$. They may be used interchangeably. Using J_L has a pedagogical advantage since, in contrast to E_L , it increases with time. Using E_L has the advantage that it is a modulus and may be applied directly in existing formulas for deflection derived in the theory of elasticity. That is why this parameter is used in many standards, for example in the common Nordic rules for the analysis of wood structures, in *Svensk Byggnorm (1967)*, and in a proposal by *Lundgren (1969)* for the analysis of board structures. In FIG. 4.1 a set of experimental creep data is presented both in terms of modulus and of compliance.

In addition to the parameters creep modulus and creep compliance, the experimental results can be presented in other ways which make the practical use of the data easier. The British Standard Institution recommends four types of diagrams for the description of total deformations in polymers (*Brandén (1970)*, *Ogorkiewicz (1970)*). The main reason why total strain diagrams are used particularly for polymers is that it is impossible to separate initial and time-dependent strain in an unequivocal way. These diagrams, therefore, ought to be used for other materials too.

The four types of diagrams are shown in FIG. 4.2. The basic diagram is (A) the *total strain vs. time diagram*. The information in this diagram is replotted in new forms in diagrams (B), (C), and (D). The influences of stress and, possibly, temperature are shown by different sets of curves.

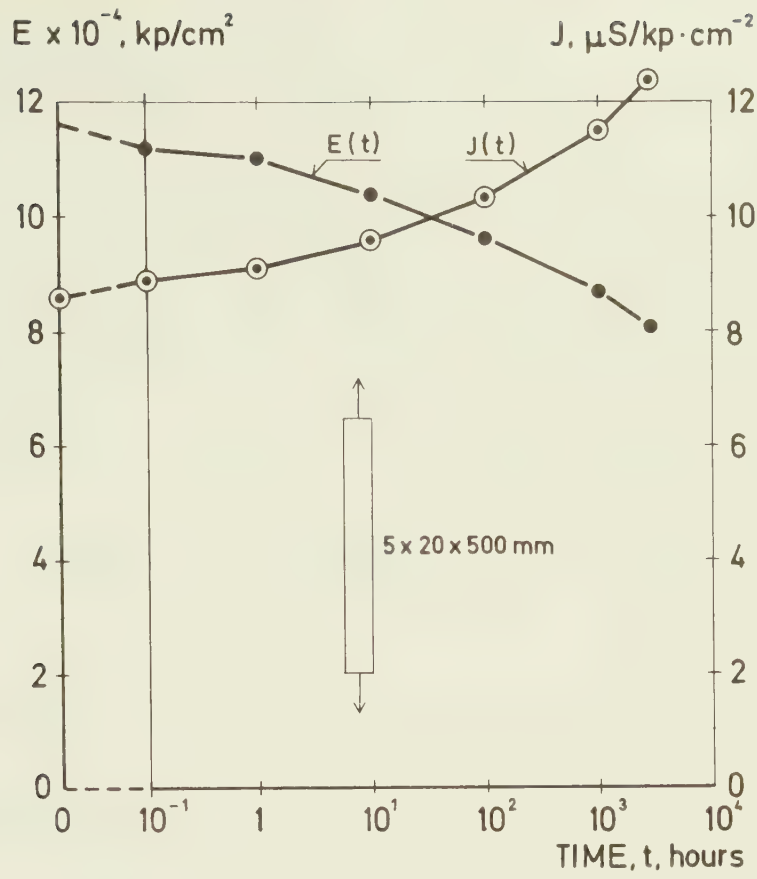


FIG. 4.1 Example of data presentation in terms of compliance J and modulus E for 5 mm U-plywood in tension ($\sigma = 100 \text{ kp/cm}^2$). The curves are based on data from *Lundgren (1968)*.

The *isometric stress vs. time curves* (B) show the time needed to reach a certain strain for various stress levels.

The reason why the *creep modulus vs. time diagram* (C) is drawn is that the modulus is a parameter in the deflection formulas in the theory of elasticity, as mentioned earlier. The curves are constructed from the original data by dividing stress by strain for each time. If the material is linearly dependent on stress you only get one curve in this type of diagram.

The *isochronous stress vs. strain diagram* (D) shows the relationship between stress and strain at different times.

It should be pointed out that several creep experiments must be carried out in order to construct diagrams like (B) and (D). Each creep experiment is represented by a horizontal line in these two diagrams.

Isochronous stress vs. strain curves indicate non-linearity between stress and strain if they deviate from straight lines. *Ogorkiewicz (1970)* considers this type of diagram to be the most useful representation of stress-strain behaviour for polymers, much more useful than conventional stress-strain-diagrams. The same can be said about the other building materials. Isochronous stress vs. strain diagrams are shown by *Rüsch (1960)* for concrete, and for cellular concrete in Appendix C, FIG. C.5. FIG. 4.3 shows an isochronous stress vs. strain diagram for a polyester-stone-composite obtained recently by *Hedin and Strand (1971)*.

Ogorkiewicz (1970) gives the following example of the use of these diagrams: components are to be produced using a polymer with the isometric stress vs. time curves shown in FIG. 4.4. The working temperature will be 40°C , and the allowable strain is 2% over 3 years. The maximum allowable tensile stress is to be determined. From FIG. 4.4 the value at 40°C can be

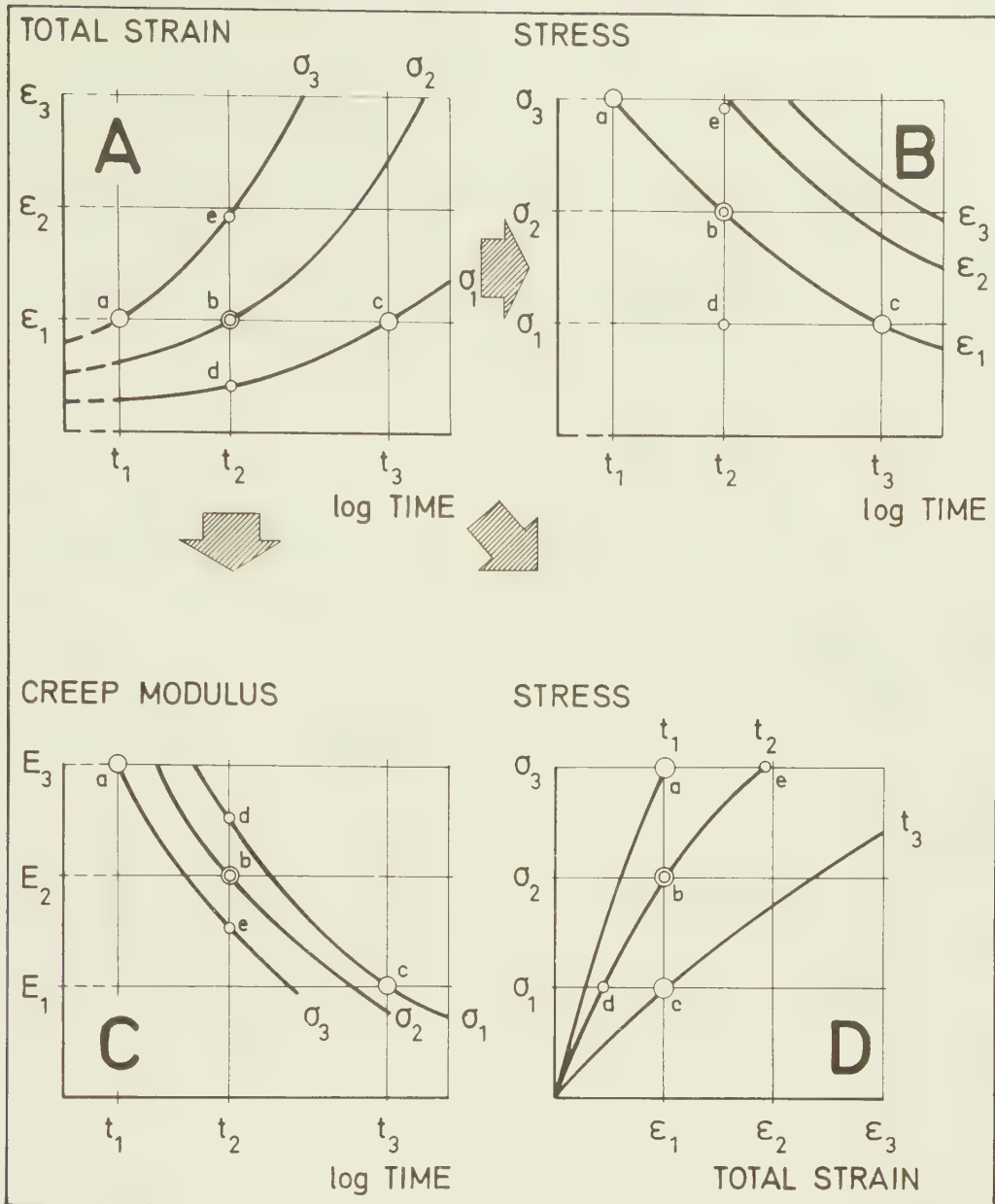


FIG. 4.2 Representations of deformational behaviour of polymers, according to *Ogorkiewicz (1970)*.

- A. Total strain vs. time, which is transformed to:
- B. Isometric stress vs. time;
- C. Creep modulus vs. time;
- D. Isochronous stress vs. strain.

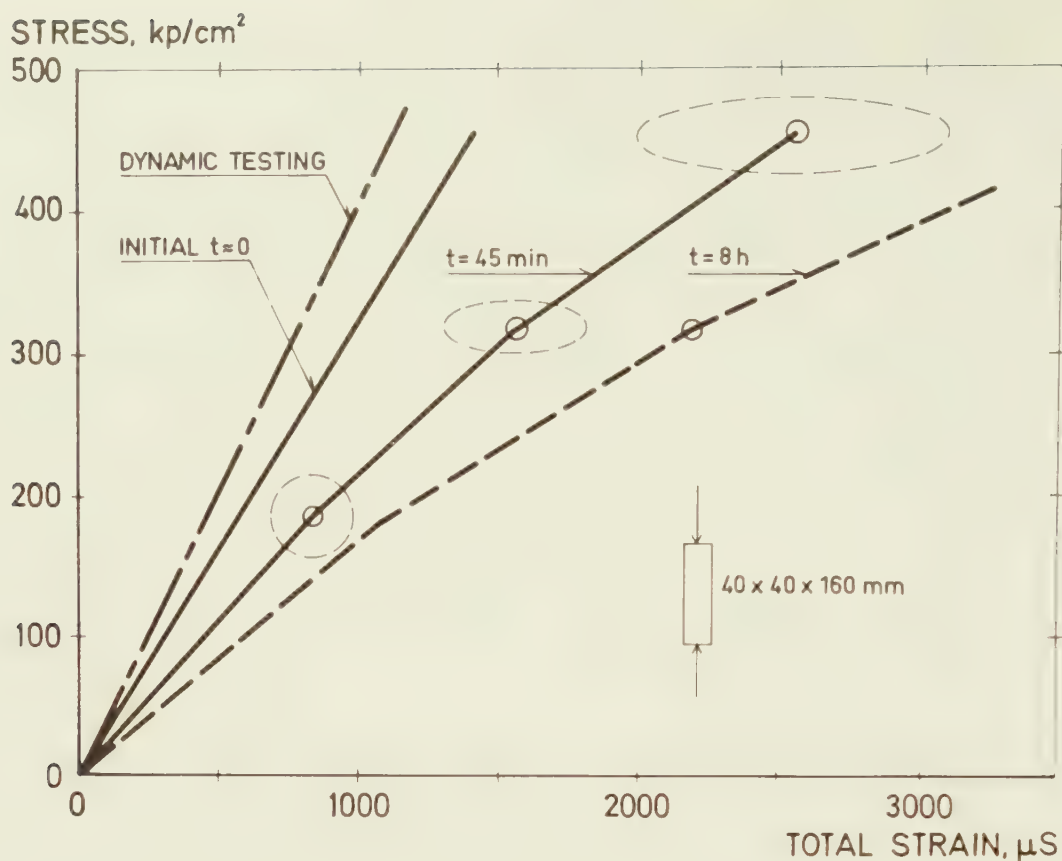


FIG. 4.3 Isochronous stress vs. strain for a polyester-stone-composite (8% polyester). From *Hedin and Strand (1971)*. The ellipses indicate the double standard deviations of the measurements.

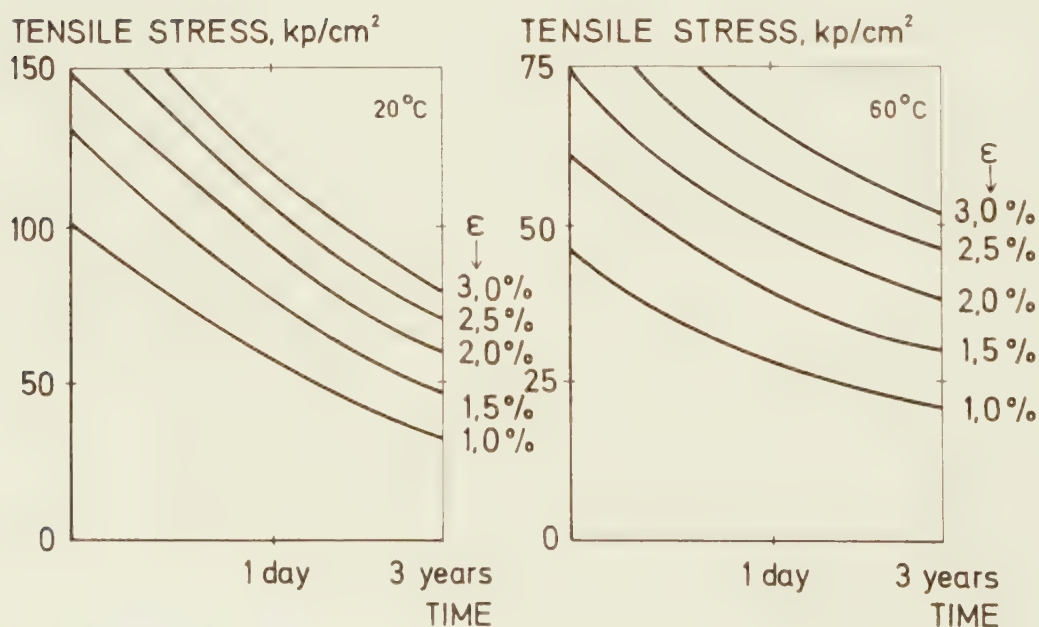


FIG. 4.4 Isometric stress vs. time for a propylene homopolymer (density 0.909 g/cm^3) at 20°C and at 60°C . From *Ogorkiewicz (1970)*.

determined by linear interpolation:

At 20°C the maximum stress is 60 kp/cm².

At 60°C the maximum stress is 39 kp/cm².

At 40°C the maximum stress is 49 kp/cm².

4.2 Parameters based on creep deformations

The creep strain, ϵ_c , is determined by the expression

$$\epsilon_c = \epsilon_{\text{tot}} - \epsilon_0 \quad (4.4)$$

(see FIG. 2.1).

A practical parameter is the creep strain per unit stress, here called the *specific creep*, ΔJ . This is the same as the difference between the compliance at the time in question, J_L , and the zero compliance, J_0 . (In the previous edition (*Nielsen (1968a)*) of the present report, ΔJ was called ϵ_c^1 .)

The final creep strain, ϵ_{ce} , is the creep strain at infinite time. It is only used in connection with concrete (*Wagner (1958)*), for which one can determine a hypothetical final strain by assuming a hyperbolic time function and determine the position of its asymptote (see TABLE 5.2). The value of this concept will be discussed in Section 5.2. The final creep strain per unit stress is called the specific final creep strain, ΔJ_e (or ϵ_{ce}^1).

A very common relative measure of creep is *fractional creep* (or *relative creep* or *creep coefficient*; German: Kriechzahl), Φ , which is the creep strain divided by the initial strain,

$$\Phi = \epsilon_c / \epsilon_0 \quad (4.5)$$

Fractional creep is very widely used because it relates creep behaviour to Young's modulus, E_0 .

This parameter can be calculated directly from measurements of deflections (δ) without any conversion to strain, because the conversion factors are the same for creep as for initial deformations,

$$\Phi = \delta_c / \delta_0 \quad (4.6)$$

This is a great advantage since it makes it possible to compare creep data obtained by different testing methods. On the other hand it is not possible to compare directly the absolute level of the creep strain without knowledge of the initial strain.

Fractional creep is a function of time, $\Phi = \Phi(t)$. Since a rather large portion of the creep of many materials has taken place after one year under load, the fractional creep after this time, Φ_y , is often taken as measure of the creep tendency of a material.

Φ_y values for some building materials are given in TABLE 8.1.

A problem connected with determining Φ shall be mentioned. It is illustrated in FIG. 4.5. According to the definition, eq. (4.5) and (4.6), the fractional creep depends on the separation of the total strain into the two parts, ϵ_o and ϵ_c . A deviation, $\Delta\epsilon_o$, in the determination of the initial strain gives opposite effects in the numerator and the denominator of the equation defining Φ :

$$\Phi + \Delta\Phi = \frac{\epsilon_c - \Delta\epsilon_o}{\epsilon_o + \Delta\epsilon_o}$$

and for $\Delta\Phi$ you get

$$\Delta\Phi = \frac{\epsilon_c - \Delta\epsilon_o}{\epsilon_o + \Delta\epsilon_o} - \frac{\epsilon_c}{\epsilon_o}$$

With $\Delta\epsilon_o = \alpha \cdot \epsilon_o$:

$$\Delta\Phi = - \frac{\alpha(1+\Phi)}{1+\alpha} \quad (4.7)$$

The relative deviation of the fractional creep becomes:

$$\frac{\Delta\Phi}{\Phi} = - \frac{1+\Phi}{\Phi} \frac{\alpha}{1+\alpha} \quad (4.8)$$

FIG. 4.6 shows the variation of $\Delta\Phi$ and $\Delta\Phi/\Phi$ for $0 < \Phi < 3$ and $\alpha = 1, 5$, and 10% . $\Delta\Phi$ is increasing

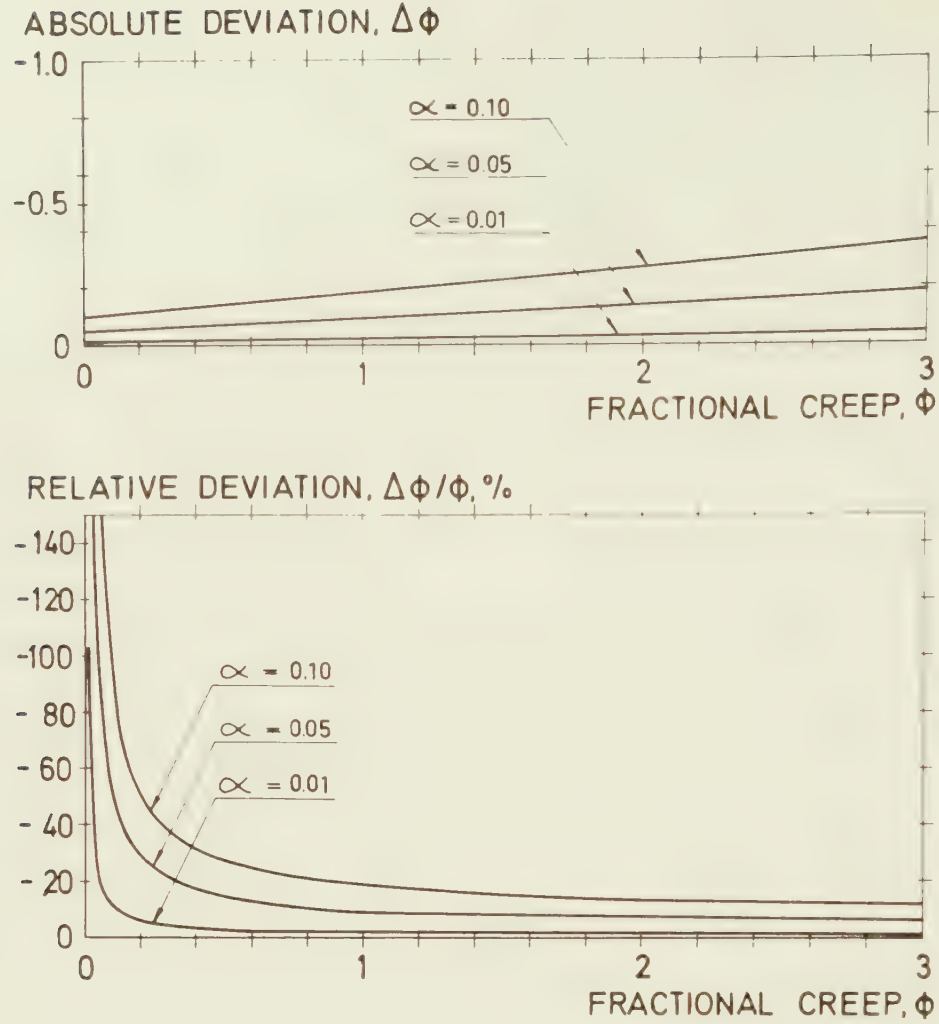


FIG. 4.6 The variation of fractional creep as a function of a variation in the initial strain of $\Delta\epsilon_0 = \alpha \cdot \epsilon_0$, $\alpha = 0.01, 0.05$ and 0.10 . Top: Absolute variation, $\Delta\phi$. Bottom: Relative variation, $\Delta\phi/\phi$.

with Φ , and $\Delta\Phi/\Phi$ naturally increases with decreasing Φ . This last fact means that one must be cautious when comparing different experimental results with small Φ values. An example of the importance of an exact determination of ϵ_0 is seen in FIG. A.5.

Fractional creep, Φ , the compliances J_0 and J_L , the specific creep ΔJ , Young's modulus E_0 , and the creep modulus E_L , are all interconnected according to their definitions. For the compliances one has:

$$J_L = J_0 + \Delta J = J_0(1+\Phi) \quad (4.9)$$

and according to eq. (4.3)

$$E_L = E_0/(1+\Phi). \quad (4.10)$$

Eq. (4.10) is commonly used in structural design, when long-term behaviour of the materials is considered.

When one wishes to study the influence of different factors on the time dependent deformations one can choose the *deformation within a certain time interval* as a parameter. An advantage of this approach is that the reference time does not need to be zero, but can be chosen at one's convenience. This method is therefore used extensively. - Analogously, one can also take *fractional creep within a certain time interval* as a parameter.

As this time interval is decreased, one will finally arrive at the *creep strain rate* (or the fractional creep rate). The creep strain rate method has the advantage of being independent of the difficulties in determining the initial strain. Another advantage is that by plotting the strain rate one can more precisely observe the influences of different factors varying with time. An example of this is seen in FIG. B.8, where the fractional creep rate has been drawn for wood beams in moisture equilibrium and

during sorption. It is quite clear when the influence of the change in moisture content is finished. - A disadvantage in using the strain rate is that its determination can be difficult, especially if the creep curve is recorded only at discrete times. Continuous recording makes it possible to calculate strain rates more accurately.

4.3 Stress parameters

In the design of machinery the term creep strength or *creep limit* is used. It was earlier defined as the stress under which no creep could occur. Eventually it was shown by accurate measurements that creep occurs in all materials at all stresses. Now the creep limit or the *creep yield limit* is defined as the stress which produces a certain strain after a certain time. This parameter is mostly used in the design of machinery with the reference values 0.2% and 1000 h. It is then written $\sigma_{0.2/1000}$.

The *creep-rupture strength* (Sw: långtidshållfastheten) is the highest stress which a material can withstand for infinite time without failure. This parameter says nothing about the magnitude of the creep.

4.4 Error propagation

In practical structural design one must know E_L or J_L in order to calculate deflections. Total deformations are seldom reported in creep literature; E_L and J_L are therefore not available. One is forced to determine E_L or J_L using data obtained in separate short-term and long-term experiments.

This method of data reporting is traditional in the field of building materials research. Young's modulus, E_0 , has been intensively studied. Creep deformations have been studied less. They are often reported alone without any mention of the corresponding Young's modulus. Thus, to get information about E_L

or J_L one is forced to combine the information taken from different types of experiments.

The determination of E_L shall be treated below as an illustration of the combinatorial procedure.

The confidence of E_L is decreased by this procedure relative to the confidence of E_L obtained directly from measurements of total deformations.

E_L can be determined from E_O and Φ by means of eq. (4.10).

$$E_L = E_O / (1 + \Phi)$$

This equation is very widely used. Φ can for example be determined using eq. (5.3):

$$\Phi = \Phi_1 t^b$$

The combination of these two equations gives

$$E_L = E_O \frac{1}{1 + \Phi_1 t^b} \quad (4.11)$$

E_O , Φ_1 and b are, according to their physical meaning, not independent of each other when they are determined from data from the same experiment. However, when these parameters are taken from quite different experiments they may be assumed to be independent of each other. Therefore, when the confidence of eq. (4.11) is to be estimated, the coefficient of variation $c\{E_L\}$ can be determined by means of the Gaussian error propagation law. One gets:

$$c\{E_L\} = \sqrt{c^2\{E_O\} + \left(\frac{\Phi_1 t^b}{1 + \Phi_1 t^b}\right)^2 \cdot (c^2\{\Phi_1\} + (\ln t)^2 \cdot s^2\{b\})} \quad (4.12)$$

$c\{E_O\}$ in this formula may be assumed to be of the same order of magnitude as the coefficient of variation of E_L determined from direct measurements. (This assumption is justified in, for example, FIG. C.7.)

In this case eq. (4.12) shows that the use of eq. (4.11) has decreased the confidence of E_L with a term including t , Φ_1 , b , $c\{\Phi_1\}$ and $s\{b\}$.

To illustrate the numerical magnitude an example is given below.

Example: The 95%-confidence limits for the creep modulus, E_L , of a material shall be calculated for two cases, one when E_L is determined directly according to eq. (4.1) and one when E_L is determined using eq. (4.11). The mean values, $\bar{E}_L$, are assumed to coincide in both cases.

In the first case the coefficient of variation is assumed to be constant $c'\{E_L\}(= 10\%)$. The 95% confidence limits are determined by

$$E'_{L,95} = \bar{E}_L(1 \pm 2 \cdot c'\{E_L\}) = \begin{cases} \bar{E}_L \cdot 1.20 \\ \bar{E}_L \cdot 0.80 \end{cases}$$

In the latter case the coefficient of variation, $c''\{E_L\}$, is determined from eq. (4.12):

$$E''_{L,95} = \bar{E}_L(1 \pm 2 \cdot c''\{E_L\})$$

The different values in the equations are assumed to be:

$$\begin{array}{lll} \bar{E}_O & = 10^5 \text{ kp/cm}^2 & c\{E_O\} = c'\{E_L\} = 10\% \\ \Phi_1 & = \Phi_y = 0.85 & c\{\Phi_1\} = c\{\Phi_y\} = 15\% \\ b & = 0.25 & s\{b\} = 0.025 \end{array}$$

(These values are close to some found for wood.)

The calculations are done for $t = 0.01, 0.1, 1, 10$, and 100 years and the results plotted in FIG. 4.7. It is seen that the indirect prediction produces broader confidence intervals, as expected. The difference between the two sets of limits increases with time.

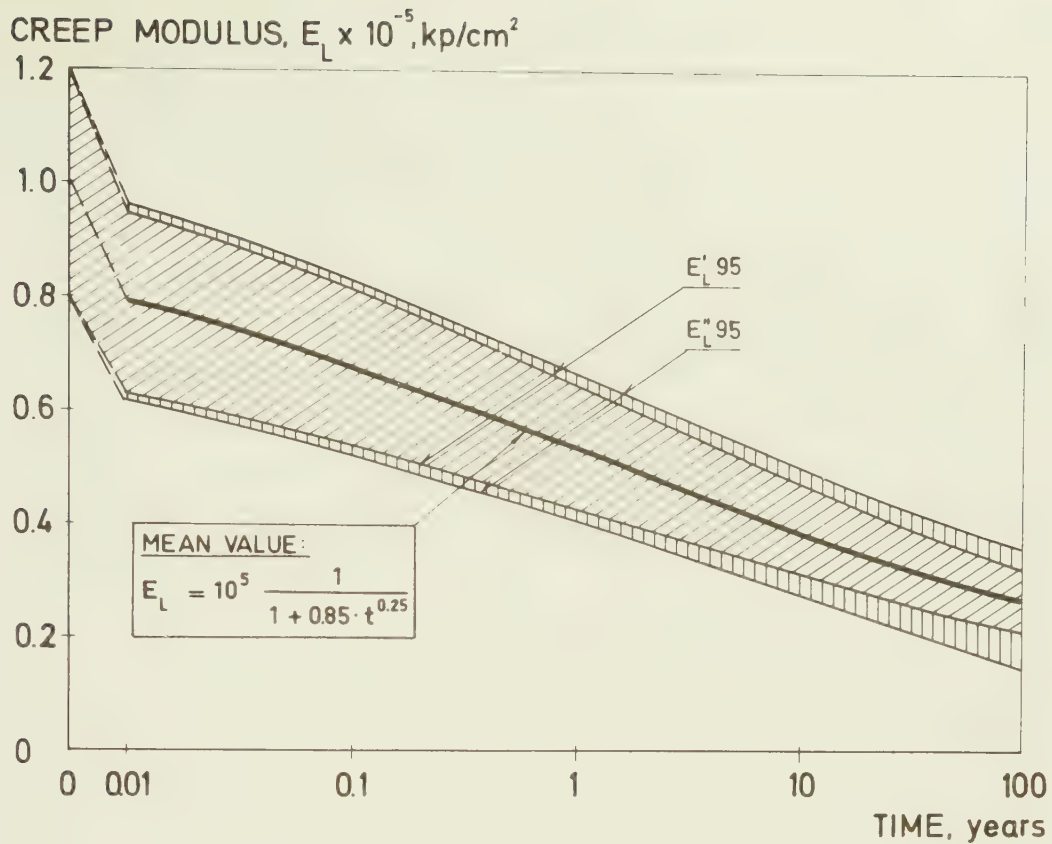


FIG. 4.7 Comparison between 95% confidence intervals for two methods of predicting E_L . A hypothetical material with $E_0 = 10^5 \text{ kp/cm}^2$, $\phi_y = 0.85$ and $b = 0.25$.

$E'_{L,95}$: Confidence curves for $c'\{E_L\} = 10\%$.

$E''_{L,95}$: Confidence curves by calculation via fractional creep. $c''\{E_0\} = 10\%$, $c\{\phi_y\} = 15\%$, $s\{b\} = 0.025$.

TABLE 4.1. DEFORMATION PARAMETERS

Parameter concerning	English	Swedish	Dimension	Letter	Definition
Total strain	Initial strain	Momentan töjning	$\mu S = 10^{-6}$	ϵ_O	
	Modulus of elasticity	Elasticitetsmodul	kp/cm^2 (N/mm^2)	E_O	$E_O = \sigma/\epsilon_O$
	Creep modulus	Långtidselasticitetsmodul	kp/cm^2 (N/mm^2)	E_L	$E_L = \sigma/\epsilon_{\text{tot}}$
	Initial compliance	Specifik töjning	$(\text{kp/cm}^2)^{-1}$ $((\text{N/mm}^2)^{-1})$	J_O	$J_O = \epsilon_O/\sigma$
	Creep compliance	Specifik totaltöjning	$(\text{kp/cm}^2)^{-1}$ $((\text{N/mm}^2)^{-1})$	J_L	$J_L = \epsilon_{\text{tot}}/\sigma$
Creep strain	Creep strain	Kryptöjning	$\mu S = 10^{-6}$	ϵ_C	$\epsilon_C = \epsilon_{\text{tot}} - \epsilon_O$
	Specific creep strain	Specifik kryptöjning	$(\text{kp/cm}^2)^{-1}$ $((\text{N/mm}^2)^{-1})$	ΔJ	$\Delta J = \epsilon/\sigma$ $= J_L - J_O$
	Final creep strain	Slutkryptöjning	$\mu S = 10^{-6}$	ϵ_{ce}	see text
	Specific final creep strain	Specifik slutkrypning	$(\text{kp/cm}^2)^{-1}$ $((\text{N/mm}^2)^{-1})$	ΔJ_e	$\Delta J_e = \epsilon_{\text{ce}}/\sigma$
	Fractional creep or relative creep	Kryptal Relativ kryptning		Φ	$\Phi = \epsilon_C/\epsilon_O$
	Creep strain rate or strain rate	Kryphastighet	$(\text{days})^{-1}$ $(\text{hours})^{-1}$	$\dot{\epsilon}_C$	$\dot{\epsilon}_C = d\epsilon_C/dt$
Stress	Creep limit	Krypgräns	kp/cm^2 (N/mm^2)		See text
	Creep yield strength	Krypsträckgräns	kp/cm^2 (N/mm^2)		See text
	Creep rupture strength	Varaktighetsgräns eller Långtidshållfasthet	kp/cm^2 (N/mm^2)		See text

In this chapter I will consider some of the expressions which can be used to represent the time function $f(t)$ in eq. (3.2). The total deformation must be split into an initial and a time dependent part before $f(t)$ can be determined. This separation will be discussed first. Then a survey of different empirical expressions is given with particular emphasis on the power function. Finally the concept of a rheological model is dealt with, as well as its practical applications.

5.1 Separation of initial deformation and creep

The distinction between instantaneous and time dependent deformation is theoretically quite clear. In practice a separation of the two is not so clear cut, since it is difficult to determine a time independent initial deformation. This uncertainty is reflected in the value for Young's modulus E_0 . It was shown in Section 4.2 how a variation in E_0 influences the calculation of fractional creep, Φ .

The reason why ϵ_0 is difficult to determine is twofold. First and foremost, the time dependent deformation takes place already while the load is being applied, since load application takes finite time. This is also the time when the rate of the time dependent deformation is greatest. Secondly, it turns out to be difficult to apply the load very fast, and to record the time dependent deformation from very short times after loading. The rate of load application is limited by the characteristics of the loading machinery. These problems will be illustrated with some examples.

FIG. 5.1 shows load and strain as functions of time in a test on cellular concrete using a pneumatic loading device built in our laboratory. The load is measured by resistance strain gauges on the loading

LOAD AND STRAIN, arbitrary units

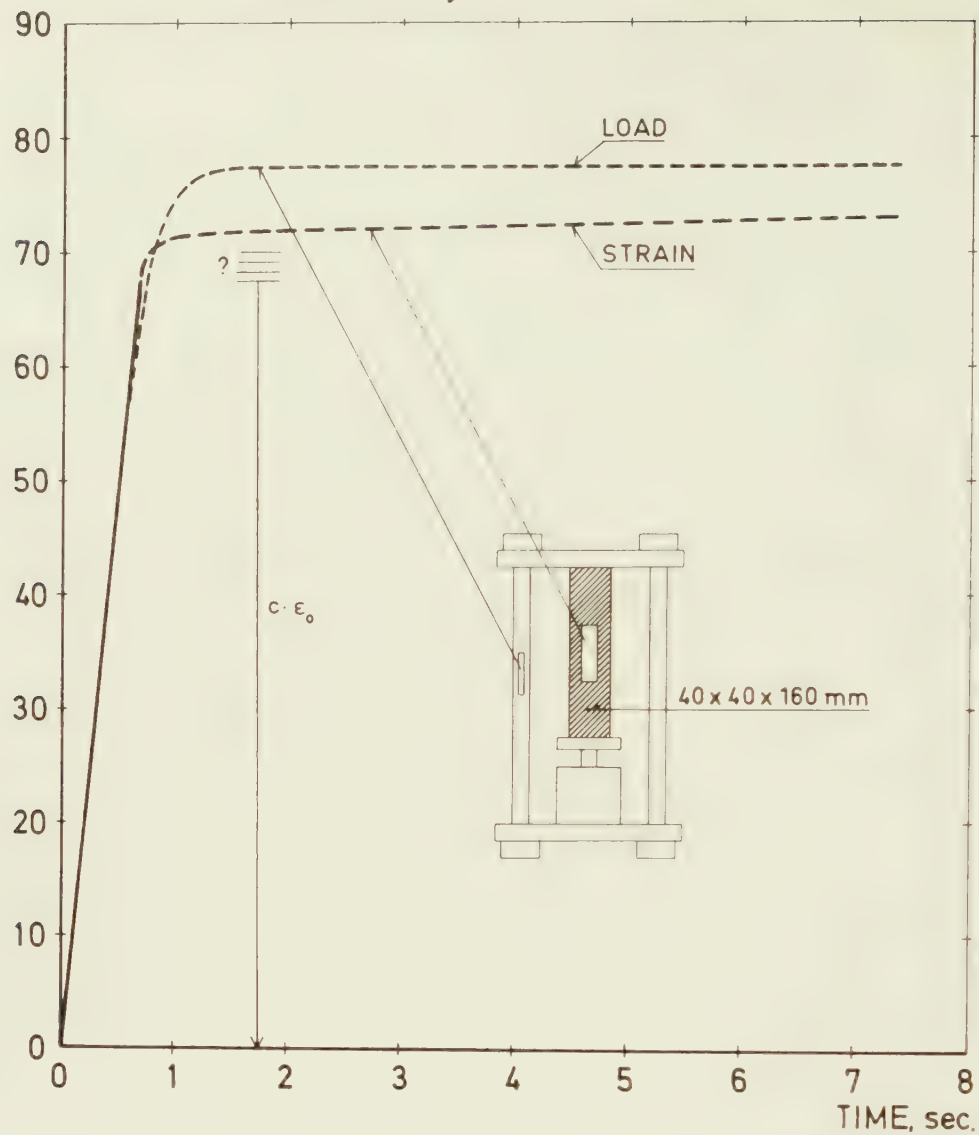


FIG. 5.1 Load and strain vs. time during loading. Cellular concrete in a pneumatic loading frame.

TOTAL STRAIN, arbitrary units

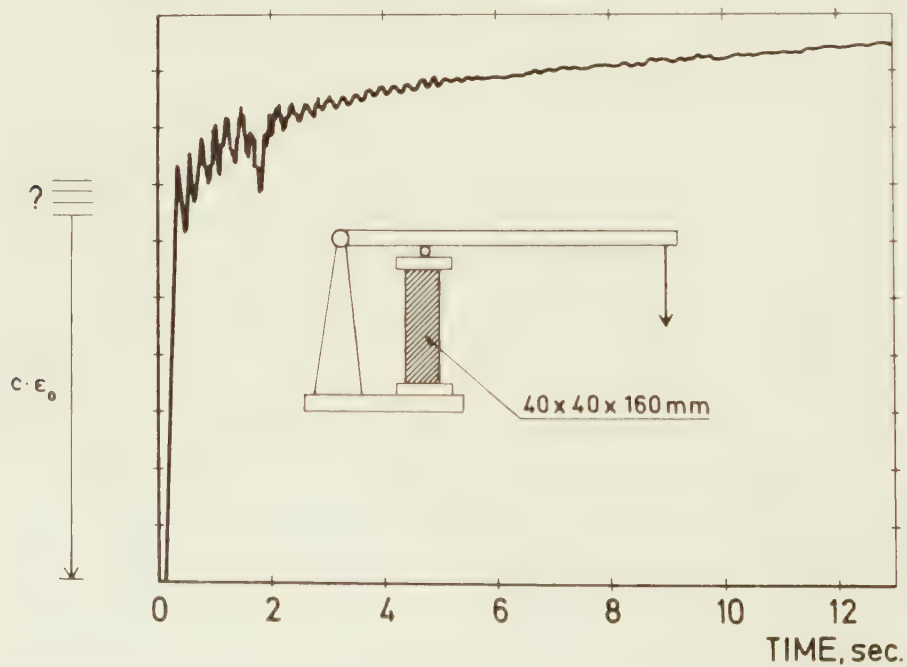


FIG. 5.2 Strain vs. time during loading. Cellular concrete in a lever machine.

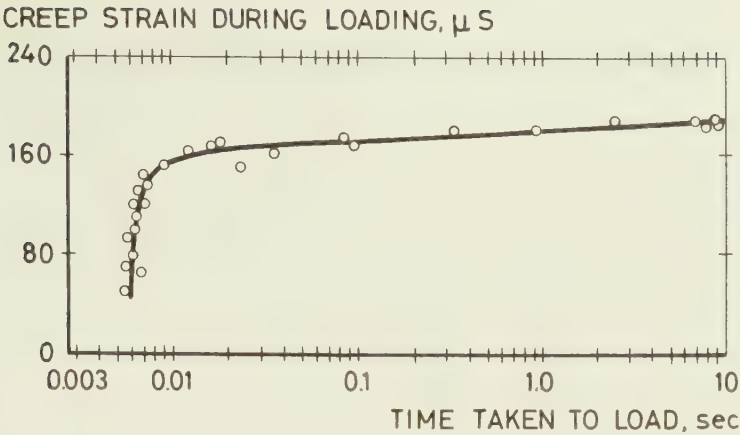


FIG. 5.3 Experimental creep data on concrete showing creep during the loading period vs. the length of this period. Each point represents one experiment. Creep is measured as total strain minus an instantaneous strain ($\epsilon_0 = 690 \mu S$) determined by an independent experiment. Concrete 1:2:3, $\sigma = 131.5 \text{ kp/cm}^2 \sim 0.63 \sigma_B$. From *Evans (1958)*.

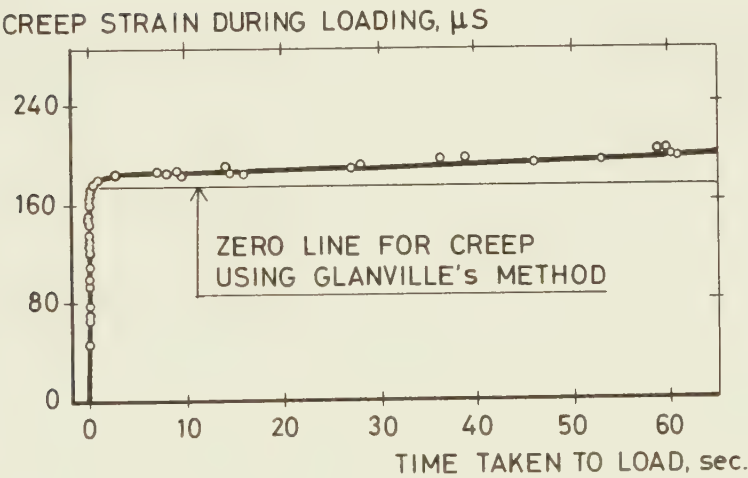


FIG. 5.4 Experimental creep data on concrete showing creep during the loading period vs. the length of this period. Each point represents one experiment. The points from 0 to 10 sec. are the same as those in FIG. 5.3. An attempt to calculate the creep during loading by extrapolation backwards has no success. From *Evans (1958)*.

frame, and the strain is measured by resistance strain gauges on the specimen. The loading curve is rounded in the transition from the linear increase to a constant value. The strain curve has a similar shape, and includes initial strain as well as some creep. Therefore, an absolute distinction between time independent and time dependent strains is not possible. The same type of curves are found using hydraulic testing machines.

When using lever machines one also sets up oscillations in the system when trying to load too fast, as shown in FIG. 5.2. This curve was also obtained in our laboratory. *Ruetz (1966)* gives a similar curve for hardened cement paste loaded in a lever machine.

Using the kind of apparatus normally employed for creep experiments one has no opportunity to study these short time effects. *Evans (1958)*, working with a special testing machine operating on compressed air, studied the deformational behaviour of concrete and stone at loading times as short as 0.005 sec. FIG. 5.3 shows creep during the loading period for one type of concrete. The creep is measured as the total strain immediately after loading minus an instantaneous strain calculated on the basis of a Young's modulus determined by a special technique on previously loaded, strainhardened material. The curve shows that at a loading time of 0.005 sec., the creep strain is about 50 μ S, or about 7% of the instantaneous strain.

The results in FIG. 5.3 are replotted in FIG. 5.4 together with other results obtained in a lever machine at longer loading times (25 to 61 sec.). A zero line for creep is also drawn. It is constructed according to a method proposed by Glanville in 1930. He obtained experimental creep curves from 5 sec. after loading, and estimated the deformation at zero time by extrapolation backwards. Evans tried this method on his own results, but FIG. 5.4 demon-

strates that it is not possible to get a good estimate of the real magnitude of ultra-short-time creep by this method.

Evans' experiment is carried out at a rather high stress level, about 63% of the crushing strength, on a rather weak concrete (compressive strength ~ 210 kp/cm²). However, the problem also appears to be present for other types of materials at lower stress levels.

Sellebold (1969) has measured recovery in flexure of small beams of cement paste. The unloading was made by burning off a thin wire, and the first reading was taken 0.20 sec. later. The dynamic modulus of elasticity was also measured, and from this a time independent instantaneous deformation was calculated. The values obtained in this way were from 0.5% greater to 8% smaller than the measured instantaneous deformations.

I obtained similar results in tests on cellular concrete (see Appendix C). The dynamic modulus of elasticity, determined by the resonance frequency method, was found to be from 0 to 14% higher than the E_0 values measured by loading and unloading.

Ruetz (1966) also discusses this problem. He does not relate the short time creep to the instantaneous strain, but to the creep after 6 days. He found that creep from 0.2 to 1 sec. and from 0.2 to 5 sec. after loading were about 1% and 3% respectively of the creep at 6 days. He therefore used 5 sec. as zero time, as shown in TABLE 5.1.

In common research practice, the initial deformation is simply defined as the total deformation at a specified time after starting the loading. TABLE 5.1 lists a number of such times from different works. This method has the advantage of giving initial deformations of the same magnitude as those obtained in a conventional stress-strain experiment, and hence

TABLE 5.1. STARTING TIME AFTER LOADING OR UNLOADING FOR MEASURING CREEP OR RECOVERY

Reference	Material	Time
<i>Sellekvold (1969)</i>	Cement paste	0.20 sec.
<i>Perkitny (1965)</i>	Wood $\perp$	3 sec.
<i>Lundgren (1968)</i>	Wood-based products	3.6 sec.
<i>Nielsen (1968b)</i>	Cellular concrete	2-4 sec.
<i>Ruetz (1966)</i>	Cement paste	5 sec.
<i>Glanville acc. to Evans (1958)</i>	Concrete	5 sec.
<i>Erickson and Sauer (1969)</i>	Wood $\neq$	1 min.
<i>Bach (1972)</i>	Wood $\neq$	1 min.
<i>Chow (1970)</i>	Furniture panels	1 min.

the result from a long-term experiment can be combined with the existing knowledge of the short-term behaviour of the materials. The modulus determined by fast loading in a creep experiment often turns out to be slightly greater than the Young's modulus determined in the slower conventional stress-strain experiment, because more creep is developed during the latter. This discrepancy can be removed if the same loading rate is chosen as that used in conventional stress-strain experiments.

The method mentioned above is chosen because of apparatus limitation rather than for its own merits. The initial deformation depends upon the loading rate, since the creep which occurs before recording starts is included in the initial deformation. The meaning of the term "initial" thus depends upon the type of apparatus. This is unsatisfactory from a scientific point of view.

More exact information about the "true" initial strain is obtained from experiments involving the propagation of stress waves through the material, or by determining the resonance frequency. These types of experiments are referred to as dynamic, and they determine a dynamic modulus of elasticity.

Some authors show that for many, especially soft, materials it has no meaning to determine an instantaneous deformation. In such cases they present the total deformation and nothing more (*Scott Blair (1949), Ogorkiewicz (1970)*).

In summary I will point out that the total deformation is the quantity which can be most accurately determined in a long term experiment. It is also the quantity which is of greatest practical importance. In "engineering" creep experiments the total strain is most often divided into an initial and a time dependent part. The separation of the two components is made at different times by different authors. This makes comparison of results obtained at short times

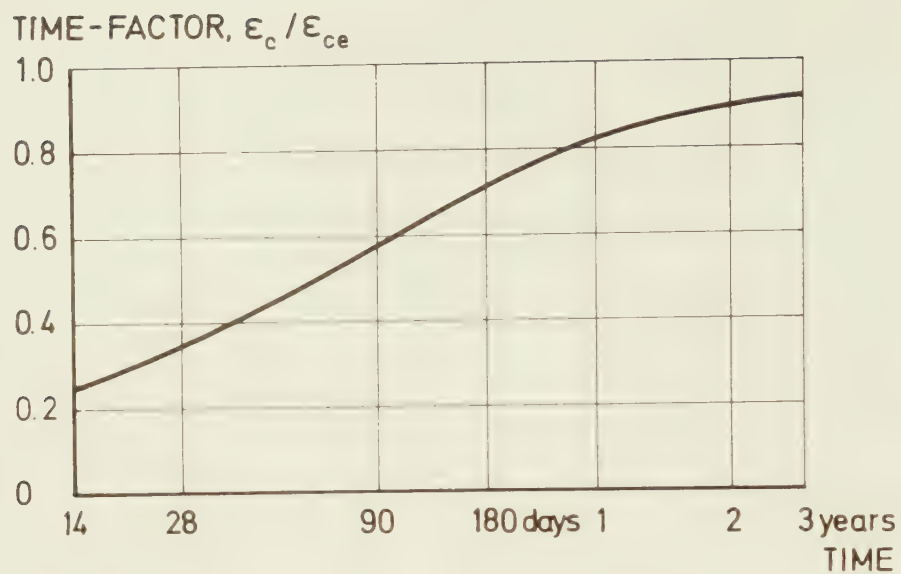


FIG. 5.5 Creep curve for concrete according to *Wagner (1958)*. The ordinate shows the creep strain divided by the final creep strain.

difficult. A method ought to be arrived at so that the initial strain would correspond to the value of the modulus of elasticity obtained in a standardized stress-strain experiment.

5.2 Time functions in general

A description of the time dependence of creep is useful for several reasons. The primary one is that it is needed in order to calculate deflections. Furthermore, a generally valid description can be used to extrapolate results from short time tests to long times. The advantage of this is obvious. A mathematical description is convenient for converting creep data to stress relaxation data.

A description of the time dependence can be made either graphically or mathematically. Diagrams are used by *Wagner (1958)* to determine a mean curve for concrete (FIG. 5.5). The many creep curves in the book by *Ogorkiewicz (1970)* on mechanical behaviour of plastics are not formulated mathematically either. A large portion of the creep results in the literature are given as diagrams only.

Although diagrams can turn out to be just as useful as formulas, the mathematical description of creep as a function of time has been very much considered in the literature. A large number of mathematical expressions have been applied successfully, which is quite natural considering the many internal and external influences on creep (see Chapter 3). *Bhattacharya et al. (1952)* mention that, for metals, more than 30 expressions are used. *Wagner (1958)* refers to 10 for concrete. The rheological models dealt with in Sections 5.4 and 5.5 represent an infinite set of time functions.

The kind of mathematical expression one chooses depends upon its intended use. As long as one's concern is the empirical description of rheological behaviour of materials it is not possible to state

TABLE 5.2. SOME EMPIRICAL CREEP FORMULAS

No.	Reference	Formula	Material
I	Andrade (1910) (a) and (b)	$L = L_0(1+ct^{1/3})_e kt$	Many metals
II	Findley et al. (1944-58) (a) and (b)	$\epsilon_{tot} = \epsilon'_0 + A \cdot t^b$	Canvas laminate Rayon Paper Plastic laminates Asbestos
	Boller (1958) (a)		Glassfiber-reinforced plastic Glassfiber-reinforced polyester and epoxy
	Chow (1970)		Furniture panels
	Clouser (1959)		Wood
III	Johnson & Frost (1952-53)	$\epsilon_c = A \cdot t^b$	Aluminium-alloys
	Weller et al. (1964)		Aluminium-alloys, normal temperatures
	Chu (1970)		Titan alloys, normal temperatures
	Cahill (1968)		Prestressing steel
	Friedrich (1950) (d)		Concrete
	Straub (1931) (d)		Concrete
	Shank (1931) (d)		Concrete
	Kajfazz et al. (1970)		Concrete
	Wittmann (1966)		Cement paste
	Schäffler (1960)		Cellular concrete
	Nielsen (1968b)		Cellular concrete
	Lundgren (1968) °)		Plywood
			Particle boards
			Fiber boards
	Kitahara & Okabe (1959)		Wood
	Ethington et al. (1965)		Wood
	Littleford (1966)		Glue-laminated beams

TABLE 5.2. SOME EMPIRICAL CREEP FORMULAS. Continued

No.	Reference	Formula	Material
IV	Johnson & Frost (1952-1953) McLean (1965)	$\epsilon_{tot} = \epsilon_o + \sum a_i t^{m_i} + \sum b_j t^{n_j}$	Metals, higher temperatures
V	Ross (1936) (d) Lorman (1940) (d)	$\epsilon_c = \frac{t}{a+ct}$ $\epsilon_c = \sigma \cdot \Delta J_e \cdot \frac{t}{n+t}$	Concrete Concrete
VI	Cottrell (1952)(b)	$\epsilon_c = \epsilon_o (\log(at+1))$ $\epsilon_c = C \cdot \log t + B$	Metals, low temperature
VII	Thomas (1933) (d)	$\epsilon_c = (a+Ce^{-Pk}) \cdot (1-e^{-rt})$	Concrete
VIII	Glanville & Thomas (1939) (d) Dischinger (1937) (d)	$\epsilon_c = \epsilon_{ce} (1-e^{-f(t)})$	Concrete
IX	Erzen (1956) according to Neville (1970)	$\epsilon_c = \sigma \cdot a \cdot \exp(c(1 - \frac{k}{t})^d)$	Concrete
X	Hansen (1960)	$\epsilon_c = k_1(1-\exp(-k(t_1-t_o)))$ $+ k \ln(t_1/t_o)$	Concrete

To the above given formulas must be added the expressions for the rheological models, (Sections 5.4 and 5.5)

Letters in parentheses refer to the references: (a) Thorpkildsen (1964), (b) Finnie and Heller (1959), (c) Altenpohl (1965), (d) Wagner (1958).
o) After transformation by the present author.

whether an expression is true or false; one can only say whether it is more or less applicable in a given situation. The extent of its applicability depends upon the exactness wanted in the situation. In some cases a rough estimate of the magnitude of creep is sufficient, in other cases more accuracy may be necessary.

The general requirements of a time function, $f(t)$, to describe basic creep or unidirectional sorption creep are that the function shall go through the origin, that it shall show a continuous increase, and that the rate of increase shall steadily decrease. Mathematically these demands can be written in the following way (*Kajfasz and Szulc (1970)*):

$$\begin{aligned} f(0) &= 0 \\ \frac{df(t)}{dt} &\geq 0 \\ \frac{d^2f(t)}{dt^2} &\leq 0 \end{aligned} \tag{5.1}$$

$f(t)$ must of course be chosen so that it fits the curve as well as is required. The fit at long times is particularly important when $f(t)$ is to be used for extrapolation to longer times.

TABLE 5.2 shows some empirical creep expressions. The original notations have been changed to conform with my usage.

Andrade's formula is given mostly for historical reasons. It was for a long time the most commonly used expression for the creep of metals. Formulas No. II, III and IV contain t to a power. The constant ϵ'_0 in formula II is not necessarily the initial strain. The power function is treated in detail in the next Section.

Andrade's formula and the power expressions all give infinite creep at infinite time. The only formulas in TABLE 5.2 which yield a final creep value are

numbers V, VII, VIII and IX, which are all used for concrete. The final creep value is determined by the level of the horizontal asymptote (see FIG. 5.5).

The concept of a final creep value (ϵ_{ce}) for concrete is practical and very commonly used (Wagner's method to determine creep of concrete is based on this concept (*Bergström and Nielsen (1969)*), but its validity is highly doubtful.

Perhaps it is possible to determine ϵ_{ce} with some certainty under constant climatic conditions, but by varying the climate it will always be possible to increase the strain. *Kajfaze and Szulc (1970)* have measured creep of concrete during normally changing relative humidity. Their specimens were kept in an underground room with no air-conditioning, and did not reach a final value even after 7 years. They applied the power function (III) to their results.

Deflections of four Swedish cantilever concrete bridges were reported by *Keijer (1969)*. The age of the bridges were from 4 to 14 years. None of the deflection curves shows a final value or even a tendency to approach a horizontal asymptote.

There appears to be no physical reason why concrete should have a final creep value. An example which indicates the lack of a limit to the creep strain of another silicate material is the following. *Humagai and Ito (1970)* measured creep on granite beams. Using viscoelastic analysis they found a viscosity of granite which was of the same magnitude as the value for viscosity which can be calculated from the movements of the upper crust of the Earth, for example from the postglacial movement of Fenno-Scandia.

5.3 The power function

TABLE 5.2 indicates that various power functions are very widely used, particularly the simple form:

$$\epsilon_c = A \cdot t^b \quad (5.2)$$

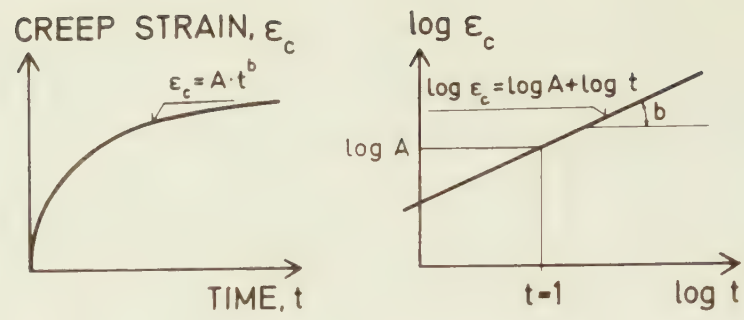


FIG. 5.6 The power function $\epsilon_c = A \cdot t^b$ is transformed to a straight line in a log-log plot.

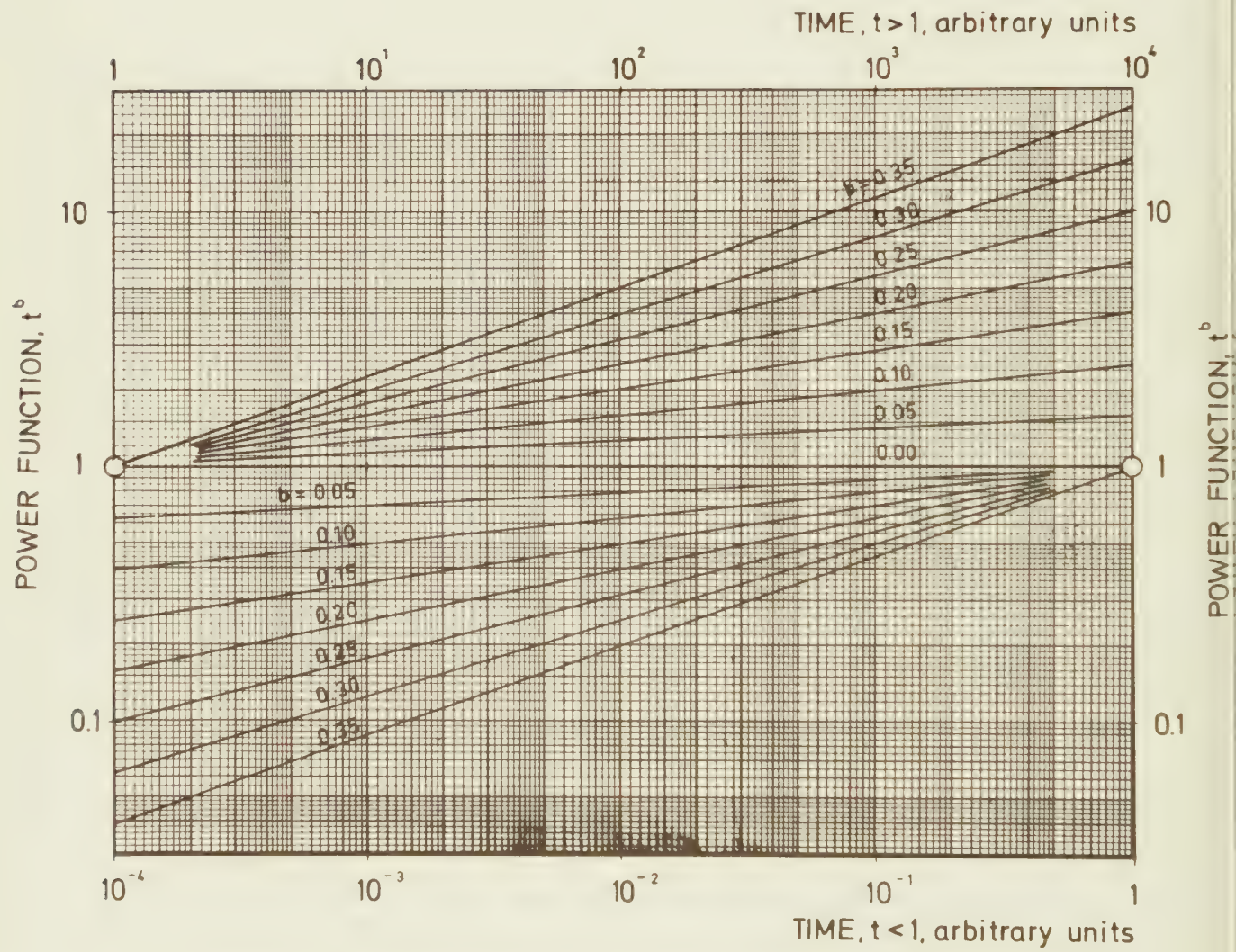


FIG. 5.7 A chart to determine the power function t^b .

For fractional creep one gets the expression:

$$\Phi = \Phi_1 t^b \quad (5.3)$$

The exponent b is in the range 0.1-0.5 (see TABLE 8.1). (For secondary creep it may reach 1.0, i.e. the creep is directly proportional to time.)

If one takes logarithms of both sides of eq. 5.2, one obtains (FIG. 5.6):

$$\log \epsilon_c = \log A + b \log t$$

Hence, a log-log plot will show a straight line with slope b and have the value $\log A(\log \Phi_1)$ at $t = 1$. This fact is of great help in evaluating creep results, and makes extrapolation of results more certain. The diagram in FIG. 5.7 can be used for graphical determination of b , as well as of values of the function t^b , when b is known.

It should be pointed out that two creep curves with the same creep rate give different b -values if the absolute magnitudes of creep differ, see for example FIG. B.7 (top, right).

FIGS. 5.8-5.10 and FIG. A.5 show some examples of straight lines in log-log diagrams. FIG. 5.8 is from my investigation of cellular concrete described in Appendix C. It is seen from this figure that the power function also describes the recovery curve for a rather long time. FIG. 5.9 is drawn using experimental results by *Lundgren (1968)* on wood-based materials.

The above mentioned examples are from experiments performed under well controlled laboratory conditions. FIG. 5.10 shows that it is possible to use the power function to describe creep of timber beams under practical conditions. The fractional creep values in the figure are computed using the results from *Nielsen, Ericsson, Gustavsson (1970)*. The symbols used in FIG. 5.10 are given in TABLE 3.2.

In FIG. 5.7 one sees that for $b = 0.10$ and for $b =$

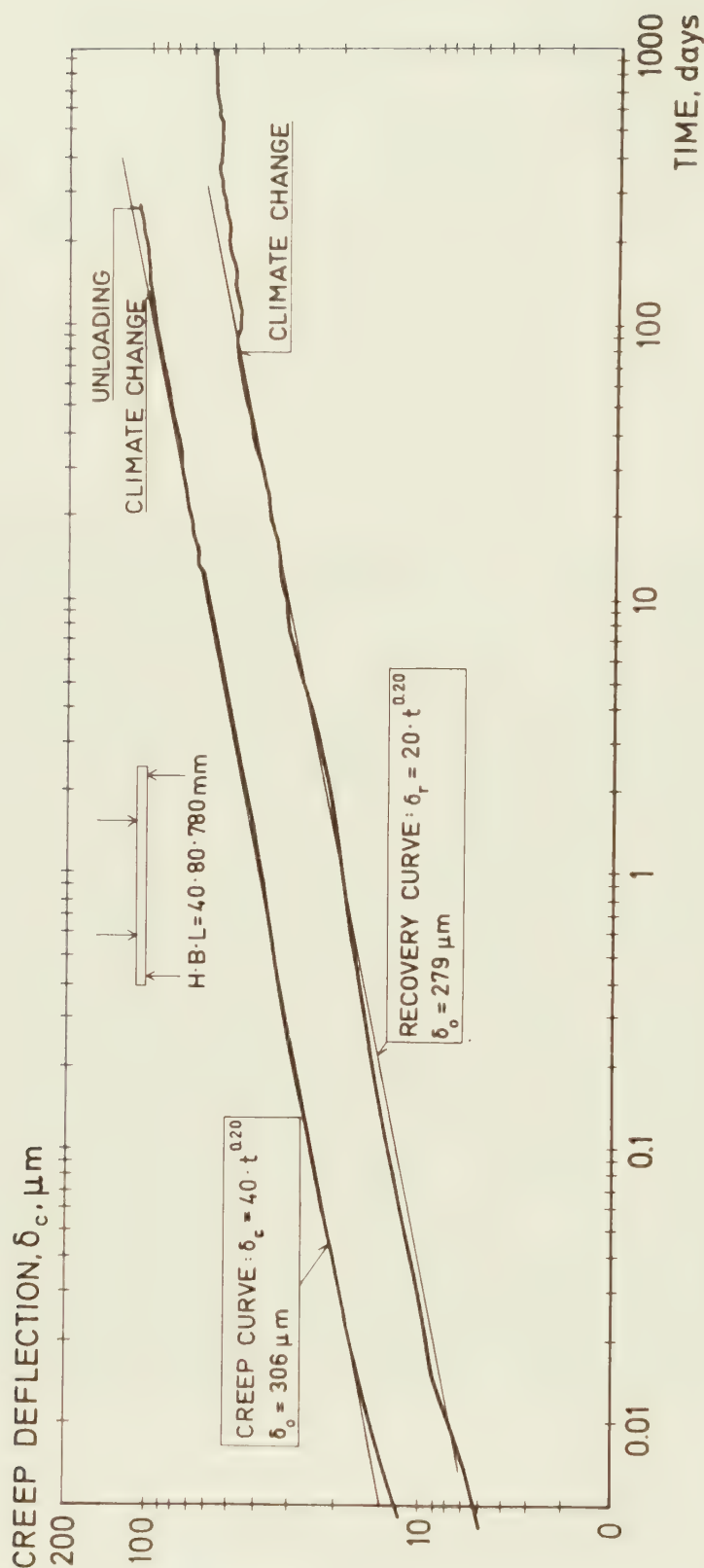


FIG. 5.8 Creep deflection vs. time for cellular concrete tested in flexure (see appendix C). The curve is linear in the log-log plot. (Factory D, $\sigma = 4 \text{ kp/cm}^2$, water saturated.) Recovery curve after unloading at 256 days. This curve is also linear for a long time.

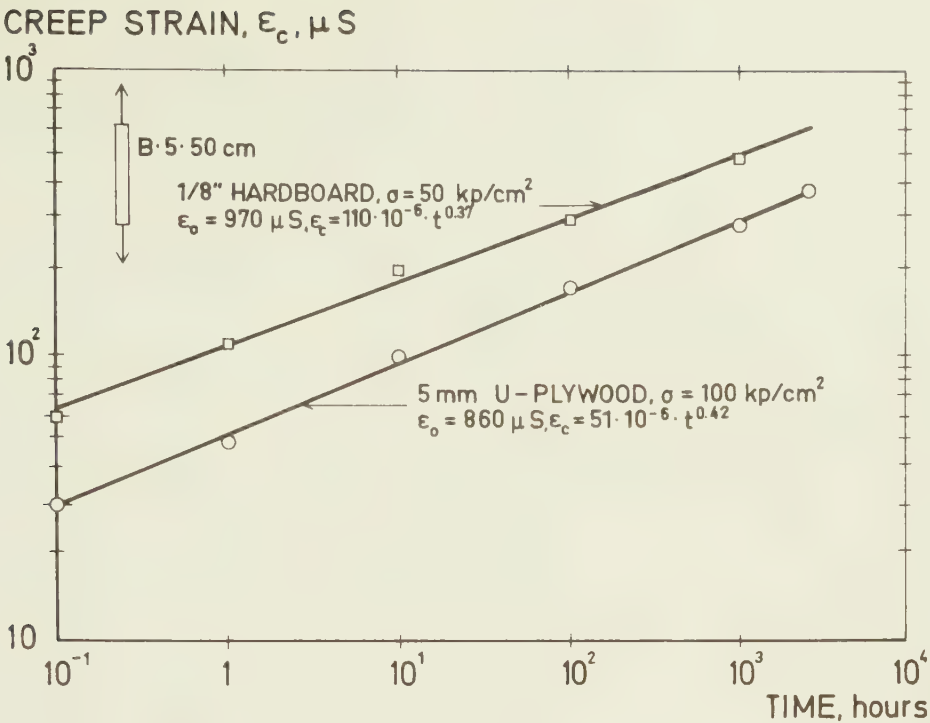


FIG. 5.9 Creep vs. time for hardboard and plywood.
From Lundgren (1968), replotted by Nielsen (1968a).

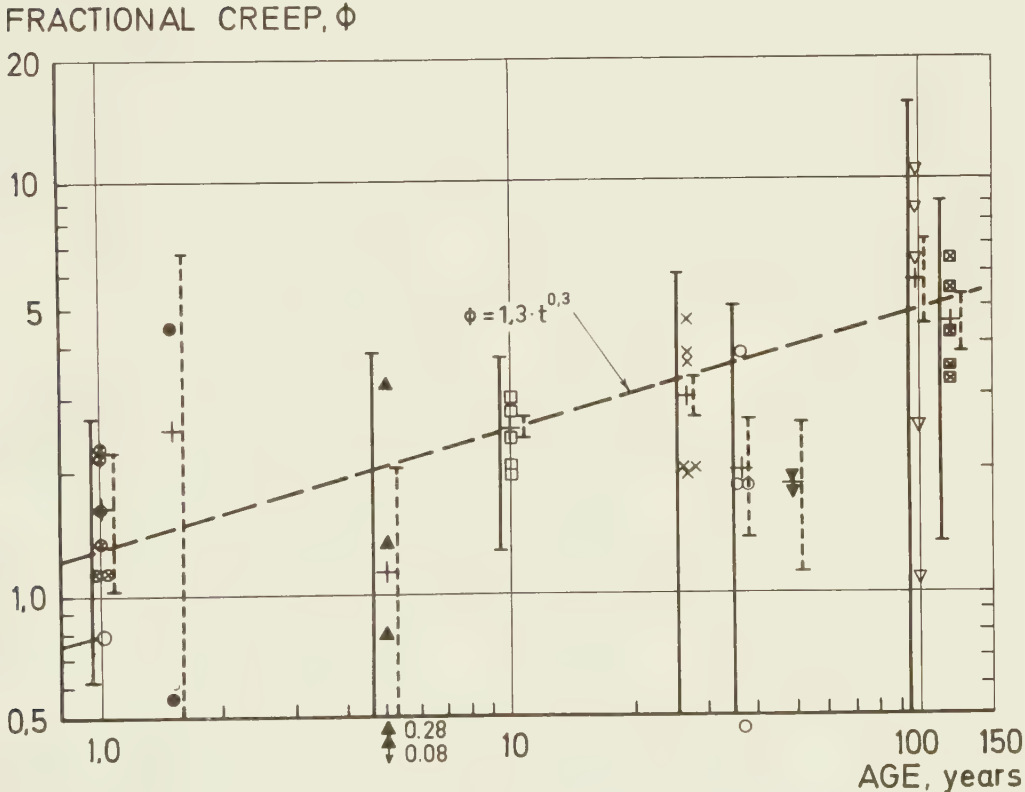


FIG. 5.10 Fractional creep values for timber in existing buildings vs. ages of the buildings.
From Nielsen, Eriksson and Gustavsson (1970). Symbols as in TABLE 3.2.
I shows 95% confidence interval for a single measurement.
I shows maximum error in measurement.

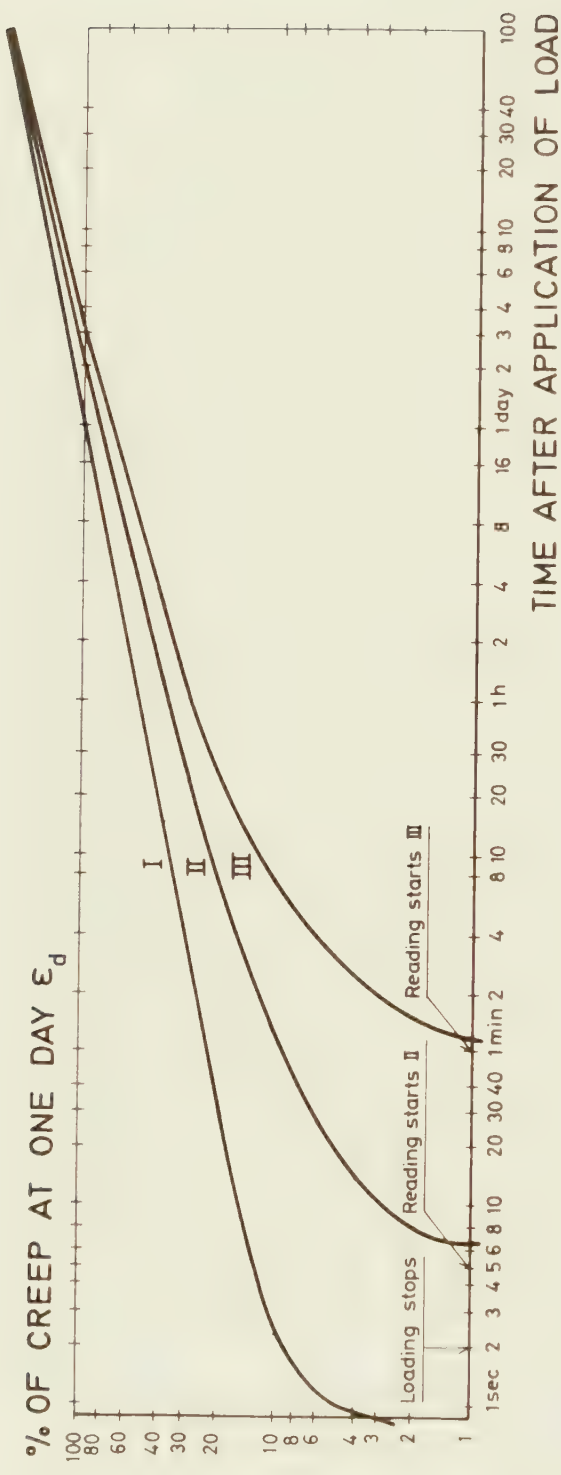


FIG. 5.11 Log-log plots for a single set of creep data, with the initial deformation, defined three different ways. The creep strain is assumed to follow a power function from zero time. The load is assumed to be applied during the first two seconds. ϵ_d is the creep after one day, according to point I.

- I. Creep calculated from the start of loading,
 $\epsilon_o = \sigma/E_o$.
- II. Creep readings taken from 5 sec. after the start of loading,
 $\epsilon_o = \sigma/E_o + 0.135 \epsilon_d$.
- III. Creep readings taken from 60 sec. after the start of loading,
 $\epsilon_o = \sigma/E_o + 0.23 \epsilon_d$.

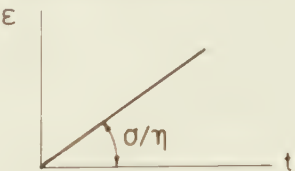
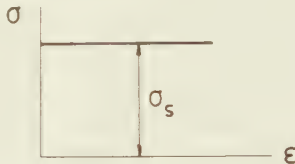
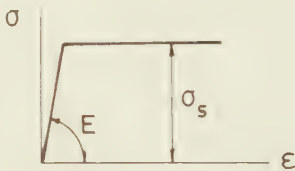
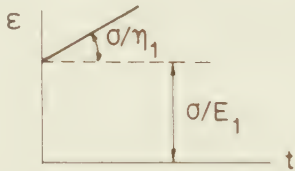
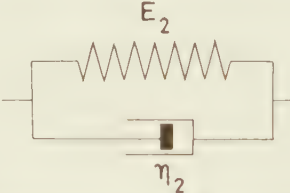
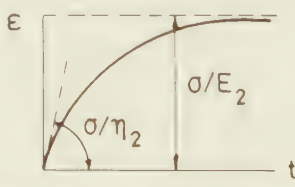
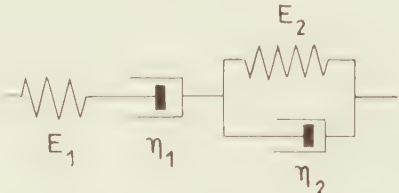
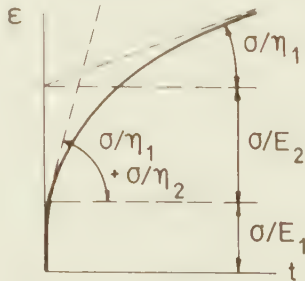
0.20 about 40% and 15%, respectively, of the creep at 1 day has occurred after 10^{-4} day = 8.6 sec. This emphasizes the fact that the appearance of the creep curve in the log-log diagram is very much influenced by the determination of the initial deformation. A further illustration will be given.

Suppose that the creep of a given material can be described by a power function with $b = 0.20$, and that there is proportionality between creep strain and stress. If the load is applied with a constant rate during the first 2 seconds and the creep is assumed to start when the loading starts, then the creep curve will have the appearance shown in FIG. 5.11, curve I. 10 sec. after the start of loading the curve is a straight line. If the first reading had been taken at 5 sec. after start of loading, then curve II would have been obtained; if after 60 sec., then curve III. The longer one waits before the first reading, the longer the curve will show a curvature. At longer times, the curves II and III will approach nearer and nearer to the "true" curve I, and after 24 hours all three are straight lines. The slopes of the curves are not the same. For curve I, $b = 0.20$, for curve II, $b = 0.21$ and for curve III, $b = 0.22$. In other words, the value of b depends upon the experimental technique.

Extrapolations are made with the greatest confidence using curve I, which has reached its proper b value already after 1 day. For the other two curves one must wait until a sufficiently long straight part of the curve has developed, for curve III 10-20 days.

The total deformation will be the same in all three cases, of course, since they are simply three ways of presenting the same data. This difficulty with the use of the log-log plot, therefore, has no influence when comparing results obtained using equal experimental methods, but influences the comparison of results obtained using different methods.

TABLE 5.3. SOME RHEOLOGICAL MODELS

Name	Code	Model	Characteristic curve
Hooke	H		
Newton	N		
St. Venant StV			
Poynting	P=H-StV		
Maxwell	M=H-N		
Kelvin	K=H N		
Burgers	B=N-H-H N		

Chow (1970), who started his readings 1 min. after loading, gives results similar to curve III in FIG. 5.9.

Finnie & Heller (1959) also point out this difficulty in applying the power functions.

Thorkildsen (1964) quotes experiments (TABLE 5.2, II) which show that b is essentially independent of stress for all the materials he treated. In my experiments with cellular concrete and wood I found that b showed no systematic dependence on stress or moisture content.

Scott Blair (1949) points out that the power function (the Notting-Scott Blair equation):

$$\epsilon_{\text{tot}} = c \cdot t^b \cdot \sigma^n \quad (5.4)$$

fairly well describes the total deformations of soft materials such as soils, butter, meat, cake, bitumens, cheese, bread and acrylic acid polymers. He also discusses the dimensional difficulty which arises when using such expressions. He is rather critical of the attempts made to divide the deformations into an initial and a time dependent part.

To summarize: The power function is applicable to many materials. It is simple to use. Its parameters depend upon the definition of the initial strain.

5.4 The simple rheological models

A rheological model of a material is a mechanical system for which the interrelationship between force, time and deformation equals that between stress, time and strain of the material.

The classical rheological models are composed of linear elastic springs, linear viscous dashpots and friction elements. (Some authors have more recently introduced nonlinear springs and dashpots, but the use of such elements is most often limited to these authors' own results.)

TABLE 5.3 shows the classical elements and some composite models. For the time independent models the stress vs. strain curves are given, and for the time dependent models the strain vs. time curves are shown. One can say that the spring is the model for the linear elastic material, the dashpot is the model for the linear viscous material and the friction element symbolizes the ideal plastic material.

The rheological models were introduced in the childhood of rheology by Reiner, Bingham and others. They intended that the use of models should help to standardize the description of the deformational behaviour of materials. *Reiner (1958)* proposed the use of the words Hooke-elasticity, Kelvin-elasticity, Burgers-elasticity etc., instead of the more ambiguous term visco-elasticity. Codes for labelling the models were introduced. The initial letter of the name was used as a code and the marks | or — showed whether the elements should be connected in parallel or in series. Some code letters are shown in TABLE 5.3. The Maxwell-model is symbolized $M = H—N$ and the Kelvin-model $K = H|N$. The combinations of elements were named after their introducers or after an outstanding rheologist.

(This method of coding was introduced for the first time by von Mises in 1930). In addition to Hooke, Newton, St.Venant, Maxwell and Kelvin, others, for example, Jeffreys ($J = N|M$), Lethersich ($L = K—N$), Burgers ($B = L—H = (H|N)—N—H$) and Schwedorf ($Schw = H—(StV|M)$) were honoured.

The behaviour of every model is characterized by a differential equation containing stress, strain, their time derivatives and the coefficients of the model. These equations can be solved using appropriate boundary conditions. For example, the solution for constant stress from zero time gives a creep expression and the solution for constant strain from zero time gives a relaxation expression. The dif-

ferential equations for the simplest time dependent models are shown below together with the expressions for creep. The constants are defined in TABLE 5.3.

Newton:

$$\dot{\epsilon} = \sigma/\eta \qquad \epsilon_{\text{tot}} = \sigma t/\eta$$

Maxwell:

$$\dot{\epsilon} = \dot{\sigma}/E_1 + \sigma/\eta_1 \qquad \epsilon_{\text{tot}} = \frac{\sigma}{E_1} + \frac{\sigma t}{\eta_1}$$

Kelvin:

$$\sigma = E_2 \cdot \epsilon + \eta_2 \dot{\epsilon} \qquad \epsilon_{\text{tot}} = \frac{\sigma}{E_2} (1 - \exp(-t \frac{E_2}{\eta_2}))$$

Burgers:

$$\sigma + \left(\frac{\eta_1}{E_1} + \frac{\eta_1}{E_2} + \frac{\eta_2}{E_2} \right) \dot{\sigma} + \frac{\eta_2}{E_2} \frac{\eta_1}{E_1} \ddot{\sigma} = \eta_1 \left(\dot{\epsilon} + \frac{\eta_2}{E_2} \ddot{\epsilon} \right)$$

$$\epsilon_{\text{tot}} = \frac{\sigma}{E_1} + \sigma \frac{t}{\eta_1} + \frac{\sigma}{E_2} (1 - \exp(-t \frac{E_2}{\eta_2}))$$

With regard to the following discussion it shall be pointed out that for the Kelvin-model the magnitude $\tau = \eta_2/E_2$ is called *the retardation time*. The magnitude $\tau' = \eta_1/E_1$ for the Maxwell-model is called *the relaxation time*.

The elements can be combined in an infinite number of ways to fit a given experimental curve. A commonly used model is obtained by putting a number of Kelvin-elements and one linear spring in series. The deformational behaviour of this model is given by:

$$\epsilon_{\text{tot}} = \frac{\sigma}{E_0} + \sigma \sum_{i=1}^n (1 - \exp(-\frac{t}{\tau_i})) / E_i \qquad (5.5)$$

Norén (1968) applied this model to the virgin creep of nailed timber joints. He used seven Kelvin elements to describe the creep curves.

The theory of *linear viscoelasticity* describes the

stress-strain relationships for models consisting of linear springs and dashpots. It can therefore be applied to the behaviour of building materials during loading times so short that the viscous strain can be assumed to vary linearly with time, and during unloading, where no viscous strains occur. An introduction to linear viscoelasticity is given by *Flügge (1967)*.

The intention of the rheological pioneers of standardizing terminology and mathematical treatment by means of the rheological models was not fulfilled. This is particularly true for building materials, because the simple rheological models fail to describe their behaviour. If one has to combine a large number of elements, then the corresponding mathematical expression is not the simplest way to describe the behaviour.

However, the rheological models have the pedagogical advantage of qualitatively illustrating the behaviour of materials very well. Particularly the Burgers body (see TABLE 5.3) shows the different types of deformations which can occur in practice, and this model is therefore very widely used as an illustration in papers and textbooks on rheology. - A further advantage of the models is that their behaviour can be simulated by an electrical circuit, where the current intensity represents the stress and the voltage represents the strain, a spring corresponds to pure resistance and a dashpot to a capacitor. Experiments with such systems are reported by *Glube (1967)* and *Winter (1970)*.

5.5 The generalized rheological models

Within the field of linear viscoelasticity there is a need for a more flexible tool to describe the behaviour of real materials than that afforded by the simple rheological models. Below I will summarize a generalized method to describe creep behaviour devel-

oped by *Nowick and Berry (1961)*.

The idea of the generalization is that the behaviour of a material can be described by combining an infinite number of Kelvin elements in a series (see eq. (5.5)). Each of the Kelvin elements is characterized by a retardation time, τ . The infinite series can therefore be described by a *distribution of retardation times*, also referred to as *the retardation spectrum*.

To determine this spectrum using experimental data is in general a difficult task, but Nowick and Berry have developed a very convenient method. They assume that the logarithms of the retardation times obey a Gaussian distribution, a so-called "lognormal" distribution. This assumption reduces the problem to finding the parameters of the distribution, namely the mean retardation time, $\bar{\tau}$, and a measure for the width of the distribution, β . In the paper by Nowick and Berry the necessary mathematical functions are solved and tabulated. The method can be applied to data showing no permanent strain.

Nowick and Berry's tabulation are made for β -values from 0 to 5. For $\beta = 0$ the distribution is reduced to the single Kelvin-element. A large value of β means a broad distribution of retardation times. Nowick and Berry report β -values of 0.6-1.2 for an alloy. The values of β for building materials are rather large. *Becker and Reiter (1970)* show broad distributions for wood, and the same is shown by *Bach (1970)*. I found β for cellular concrete to be 5.7 (see example below).

Nowick and Berry point out that considering the atomic structure of materials it is quite reasonable to expect the deformation to be characterized by a distribution of retardation times.

FIG. 5.12 shows the application of the Nowick and Berry method to recovery curves for cellular concrete

(see Appendix C). The curve fitting is good at longer times, but not so good at shorter times. This implies that the lognormal distribution does not describe the recovery curve over the entire time range. Generalized rheological models are used as a tool in research on the structural causes of deformational behaviour (*Sellekvold (1969)*).

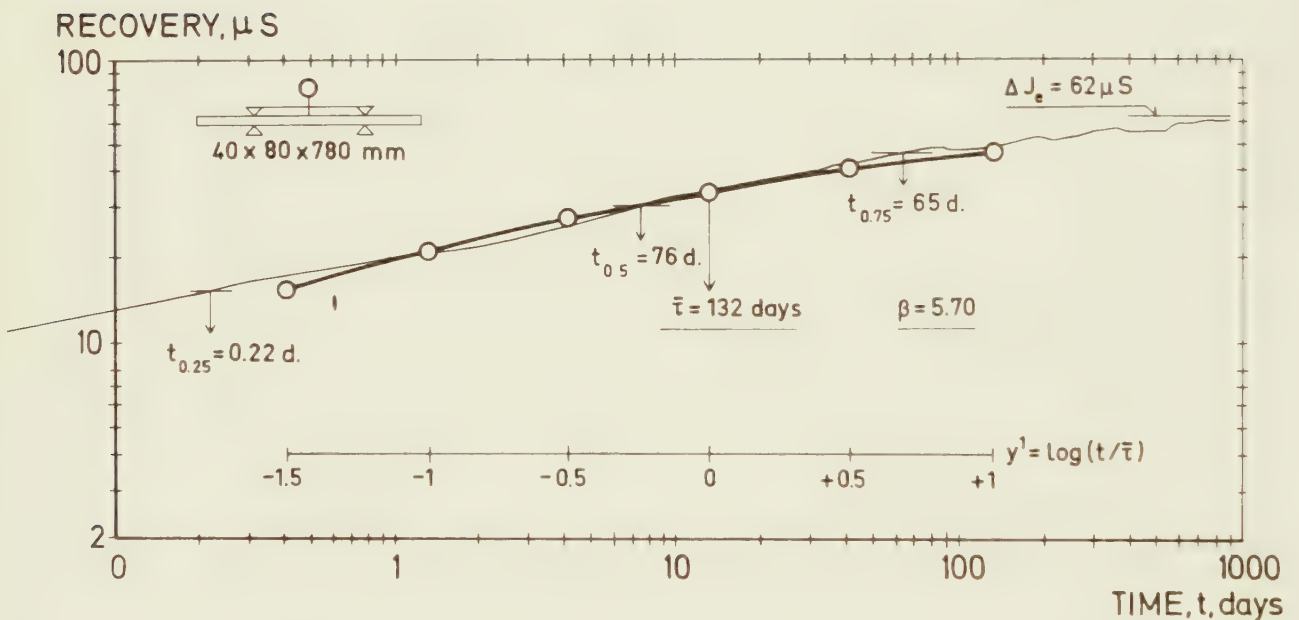


FIG. 5.12 An example of application of the curve fitting technique by *Nowick and Berry (1961)*, by which a distribution of retardation times can be estimated. Recovery of cellular concrete loaded in flexure ($\sigma_b = 4 \text{ kp/cm}^2$) for 256 days before unloading. Water saturated Dalby Siporex, $\epsilon_{or} = 279 \mu S$. Notation according to *Nowick and Berry*.

This chapter deals with the influence of stress and stress variations on creep. The influence of stress on virgin creep is dealt with first. In Section 6.2 the strain components are discussed. The Boltzmann superposition principle is presented in Section 6.3. Section 6.4 shows graphical superposition for a strain hardening material. Section 6.5 gives a mathematical formulation of superposition using the strain analysis in Section 6.2.

6.1 Stress and virgin creep

Virgin creep does not in general vary linearly with stress. Descriptions of stress dependence using sinh and power functions are treated below. The extent to which a material may be considered to be "linear" is also discussed.

Freudenthal (1950) has constructed a model for a material consisting of minute particles (molecules) and shown that for his model the relationship between stress and strain rate is as follows:

$$\dot{\epsilon}_{ct} = K_1 \sinh(K_2 \sigma) \quad (6.1)$$

K_1 and K_2 are experimental constants. According to *Scott Blair (1949)* this equation was shown to be valid by Nadai on an empirical basis.

Thorkildsen (1964) points out that eq. (6.1) has been shown to be valid for metals by Kauzmann in 1941, and for fabrics, thermosetting plastics, asbestos and glassfiber-reinforced plastic by Findley et al. (1951). *Kingston and Clarke (1961)* applied it to wood. *Wittmann (1967)* has shown the validity of this equation for cement paste in compression as well as in tension. When the creep strain at a fixed time and temperature is considered, eq. (6.1) may be integrated to yield:

$$\epsilon_{ct} = K_3 \sinh(K_2 \sigma) \quad (6.2)$$

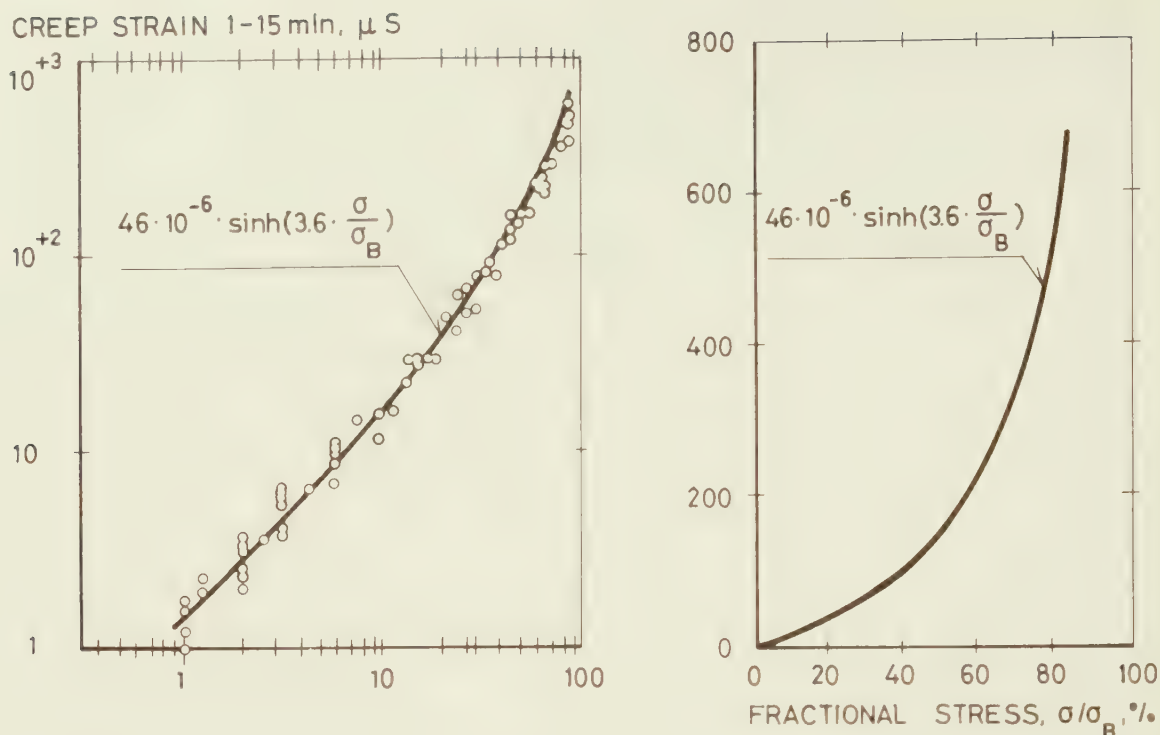


FIG. 6.1 Creep of concrete in compression vs. fractional stress, σ/σ_B . Left: Log-log scales. Right: Linear scales. Data from *Evans (1958)*, $\sigma_B = 210 \text{ kp/cm}^2$. The sinh-curve applied by the present author.

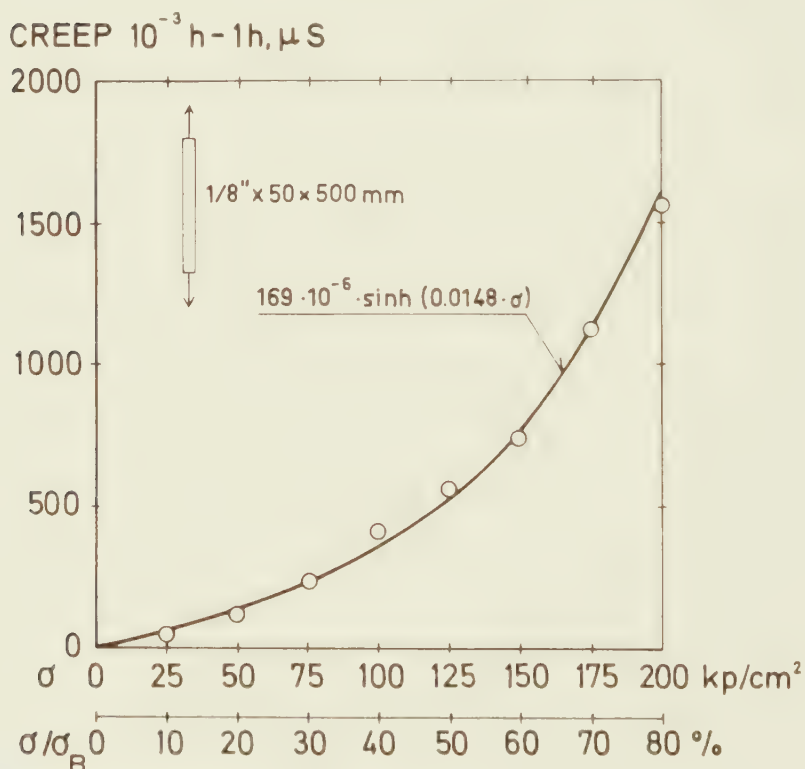


FIG. 6.2 Creep from 10^{-3} hours to 1 hour of 1/8" fiberboard in tension vs. stress, σ , and vs. fractional stress, σ/σ_B . Data from *Lundgren (1968)*. (Masonite at 65% RH.)

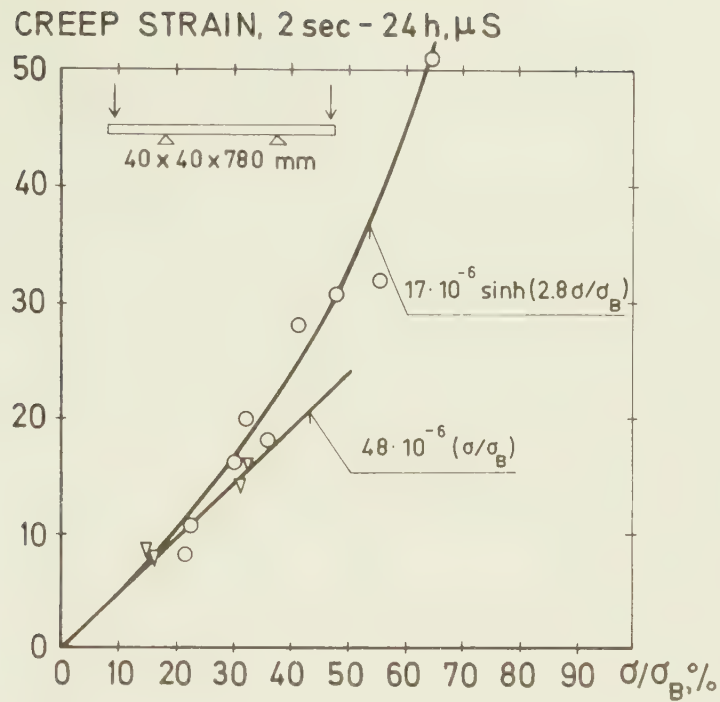


FIG. 6.3 Creep of cellular concrete vs. fractional bending stress, σ/σ_B . Results from two sample sets chosen at random from the same factory (Dalby Siporex). 43% RH. $\bar{\sigma}_B = 11 \text{ kp/cm}^2$. (Nielsen (1968b)).

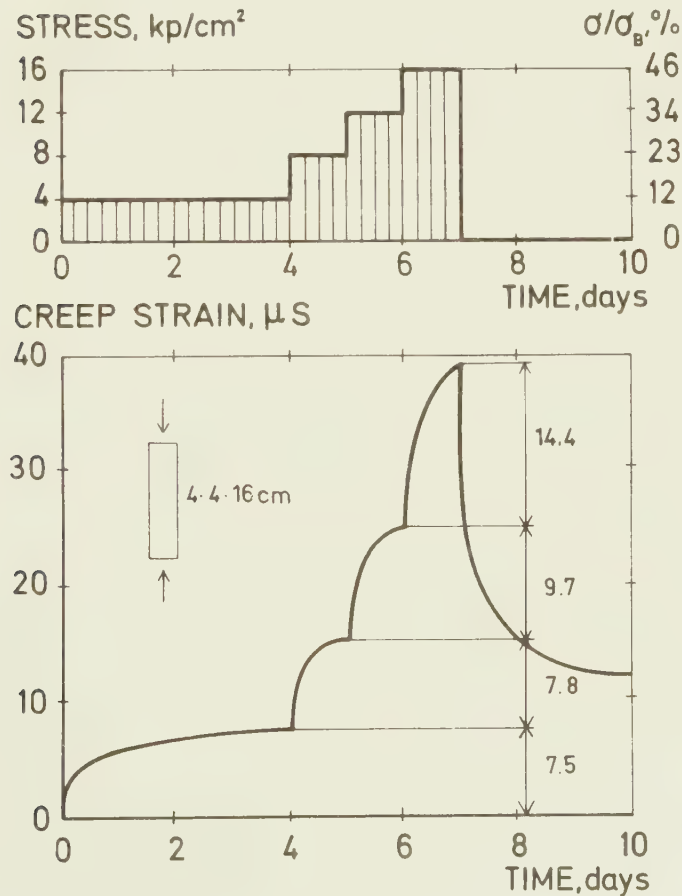


FIG. 6.4 Creep of cellular concrete in compression at different stress levels. Södertälje Siporex, 43% RH, 20°C , $\bar{\epsilon}_0 = 174 \mu S$. The curve is mean of two experiments.

K_3 is a parameter which depends upon the material, time and temperature. To illustrate the validity of eq. (6.2) some examples are given below.

Some experimental results on concrete, obtained by *Evans (1958)*, are shown in FIG. 6.1. The materials are the same as those referred to in FIG. 5.3. I have applied the sinh equation to the results, and the fit is seen to be fairly good. (The original curve-fitting by Evans is shown in FIG. 6.5.) *Wagner (1958)* and *Neville (1970)* refer to further results on concrete which show the same general tendency as that found by Evans.

FIG. 6.2 shows experimental creep results on fiber-board taken from *Lundgren (1968)*. The data are plotted in a stress vs. strain diagram. This plot is chosen as being representative of data on several other wood based materials at several humidity conditions in Lundgren's thesis. I have applied a sinh equation to the results.

Data from my own investigation of cellular concrete (*Nielsen (1968b)*) is plotted in FIG. 6.3. The scatter is rather large, because the samples were chosen at random from normal production runs, but the trend is the same as in the previous figures.

The nonlinearity between stress and creep strain can also be shown in experiments where the stress is increased in steps on a single sample. The result of such an experiment is shown in FIG. 6.4, which shows creep of cellular concrete in compression. (A lever machine was used, and the strain was recorded by resistance strain gauges.) There is nonlinearity at all stress levels, but it is most pronounced at the 46% level. Similar results are given for a polyester-stone composite by *Hedin and Strand (1971)*. (FIG. 4.3 is from this investigation.)

The proportionality between creep and stress is said to be proved in many investigations of creep at working stress levels.

This result is not a contradiction of the sinh rule just discussed. For small values of x one has $\sinh x \sim x$. As seen in FIGS. 6.1 to 6.3, it is possible to use a straight line up to a stress level (i.e. σ/σ_B) of about 30%, since the deviation of the sinh curve from a tangent through the origin first becomes considerable above this level. The many creep curves for concrete quoted by *Wagner (1958)* also show linearity up to about 30% of the ultimate strength.

This observation, and the fact that creep data usually displays great scatter, makes the hypothesis of a linear relationship between virgin creep strain and stress at common stress levels acceptable. Eq. (6.2) may then be simplified:

$$\epsilon_{ct} = K_3 \cdot K_2 \cdot \sigma = a \cdot \sigma \quad (6.3)$$

Though the sinh equation seems to be very well supported by experiments, it is not used in design calculations, surely because of its complexity. In design, linearity is assumed whenever possible, and if this fails the simple power equations are substituted. At a certain time and temperature one may write:

$$\epsilon_{ct} = B \cdot \sigma^n. \quad (6.4)$$

B is a parameter that depends on the material, time, and temperature. The exponent n is larger than or equal to one. If n equals one, then eq. (6.3) and eq. (6.4) are identical.

The value of n can be evaluated graphically by plotting the test results in a log-log diagram. n is the slope of the resulting line (see FIG. 5.6).

Eq. (6.4) was first applied (to steel alloys) by Norton in 1929. It is therefore also called *Nortons law*. It is commonly used for metals, particularly in the temperature range from 40% to 70% of the melting temperature. It probably also is appli-

cable to some polymers. *Scott Blair (1949)* uses the power function to describe the total deformation of soft materials.

Evans (1958) has applied power functions to the results on concrete given in FIG. 6.1. His curve fitting is shown in the log-log plot of FIG. 6.5. He found two n -values. The first one was $n = 1.0$, which means linearity. This holds up to about 30% of the crushing strength (see the discussion of linearity above). The second n -value of $n = 1.95$ was valid at higher stress levels.

Power functions have also been applied to creep of wooden beams by *Sawada (1957)*. He obtained $n = 3$ or 4.

The stress dependence of the fractional creep shall now be treated. The fractional creep at a certain time Φ_t , can be written in the following way, according to eqs. (4.5) and (6.2):

$$\Phi_t = \frac{\epsilon_c}{\epsilon_o} + \frac{K_3 \cdot \sinh(K_2 \sigma)}{(\sigma/E_o)}$$

This expression can be transformed into

$$\begin{aligned} \Phi_t &= \frac{K_2 \cdot K_3 \cdot E_o \cdot \sinh(K_2 \sigma)}{K_2 \cdot \sigma}, \text{ or} \\ \Phi_t &= a E_o \frac{\sinh(K_2 \sigma)}{K_2 \cdot \sigma}. \end{aligned} \quad (6.5)$$

The last term in eq. (6.5) equals unity for small values of $K_2 \sigma$. In this case

$$\Phi_t = a \cdot E_o \quad (6.6)$$

At higher stress levels the last term is larger than unity and Φ_t increases with increasing stress as shown in FIG. 6.6. The stress at which Φ_t deviates appreciably from the straight line is about 30% of ultimate stress as mentioned above.

Examples of nonlinearity between Φ and σ are given in

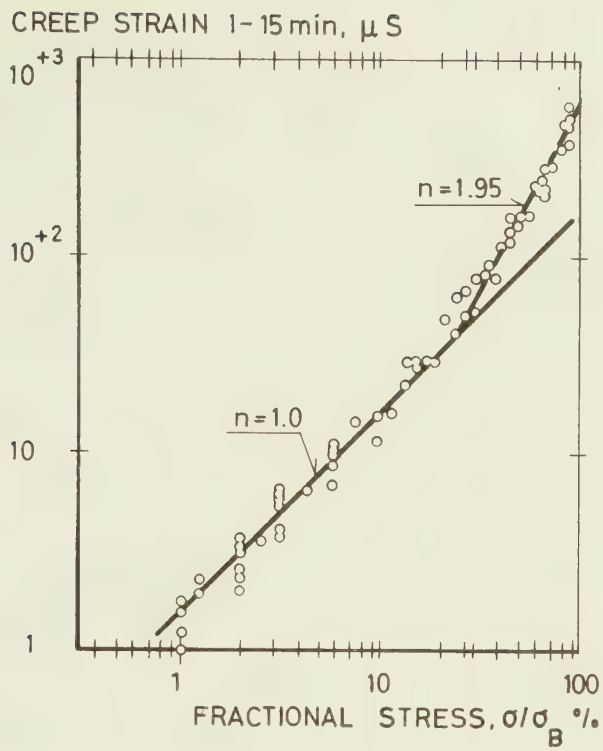


FIG. 6.5 Creep of concrete. Exactly the same results as in FIG. 6.1, but curve fitted using power-functions. From *Evans (1958)*.

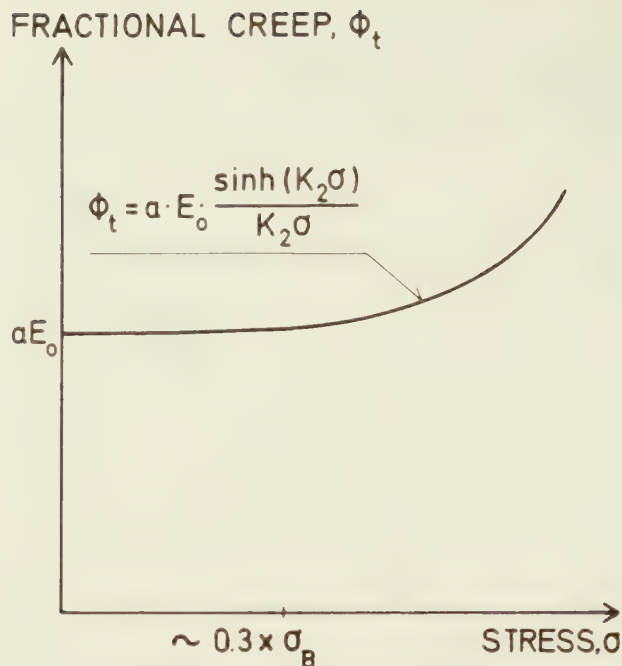


FIG. 6.6 The stress dependence of the fractional creep ϕ_t . The stress value at which ϕ_t deviates from a straight line may depend on the material.

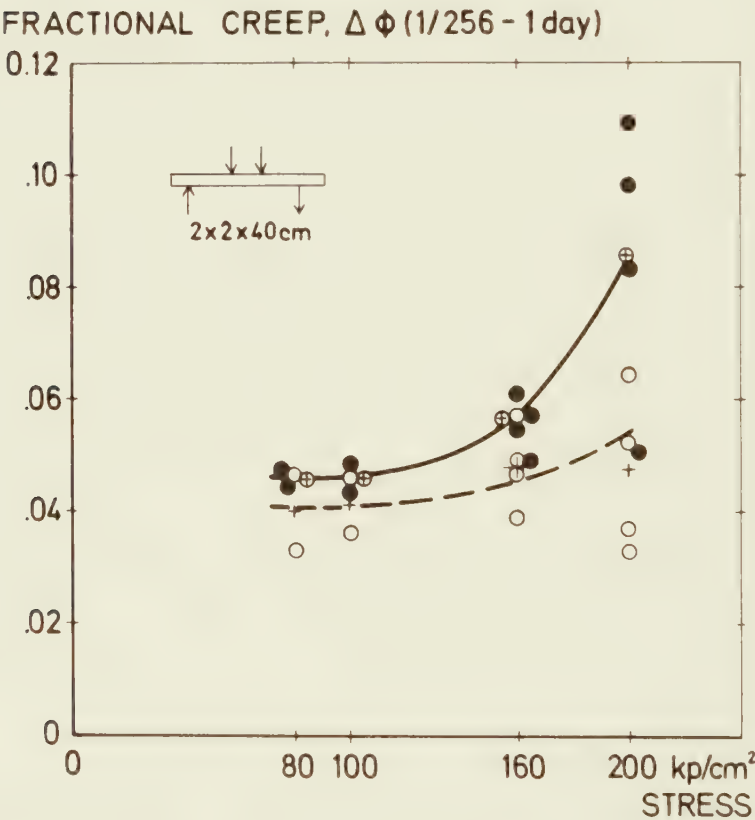


FIG. 6.7 The influence of stress and relative humidity on the fractional creep from 1/256 to 1 day for pine and spruce. Data from *Cederberg-Danielsson (1970)*, replotted by the present author. See Appendix A.

	Single	Mean	Trend
Symbols: 64% RH	○	+	— —
80% RH	●	⊕	——

FIGS. 6.7, 6.11, A.3 and A.4. FIGS. 6.7, A.3 and A.4 are from our investigation of creep of wood described in Appendix A.

In summary it shall be said that for many materials linearity between stress and virgin creep strain may be assumed up to about 30% of the ultimate strength. In practical design linearity is often assumed up to 50-60% of the ultimate strength. This may be an acceptable hypothesis because the scatter of creep data often is rather large. Where linearity may not be assumed, sinh-functions or power functions may be used to describe the relationship between creep strain and stress.

The whole range from zero to high stress levels is best described by the sinh functions.

6.2 Strain components

The components of the total strain for a hypothetical building material are shown in FIG. 2.2. The relationship between the different components may be either stress independent or stress dependent. I will call attention to some general principles concerning the strain components and their interrelationships. These principles may be used to establish rules for superposition.

The principles are summarized by the following statements:

- *1 The delayed elastic strain reaches a final value after a certain loading time.
- *2 The delayed elastic strain varies linearly with stress at all stress levels.
- *3 The final delayed elastic strain always amount to a certain proportion to the instantaneous elastic component at unloading, for a certain type of material in a certain climate.
- *4 The delayed elastic strain recovers more slowly than it develops.

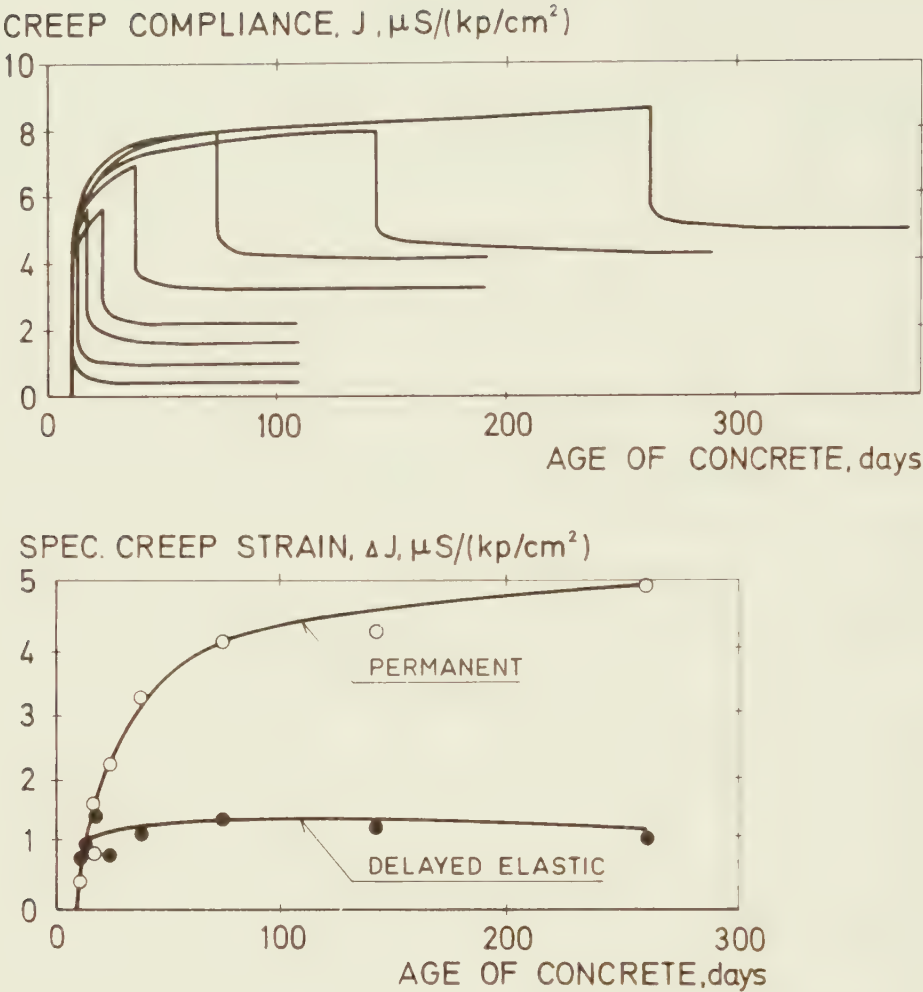


FIG. 6.8 The components of time dependent strain in concrete. From *Illston (1965)*. Top: The actual experimental curves. Bottom: Evaluation of the delayed elastic and permanent parts of the strain. Compression stress between 70 and 85 kp/cm^2 .

- *5 The viscous component of strain increases with time at a decreasing rate, but it never stops. This is true at least as far as virgin creep is concerned.
- *6 The viscous component of strain increases more than linearly with increasing stress.

These principles are based on studies of phenomenological rheology and on investigations of specific materials such as concrete, cellular concrete, wood and wood-based products. The proposed principles are assumed to be valid for all building materials in accordance with the axiom in mechanics that all materials exhibit all properties to greater or lesser extents (*Schniewind (1968)*). The principles have not been tested systematically for all the materials in experiments designed for this purpose, but they may be illustrated by examples taken from other studies of the above mentioned materials.

*1 The delayed elastic strain develops rather fast in the beginning. After a certain time (which certainly is a characteristic of the material under consideration), it reaches a final value and remains at this value during the rest of the loading period. (Upon unloading it recovers fast in the beginning and then slows down, but it can continue to recover for a longer time than the loading lasted.)

A most suitable experiment, which illustrates this principle, was done by *Illston (1965)* on concrete. (Experiments similar to Illston's were made by *Glucklich and Ishai (1961)*.) Some of Illston's test results are shown in FIG. 6.8. Concrete specimens were loaded at the age of 10 days after casting, and unloaded at different times. The creep compliances are shown in the top diagram and the specific creep strain, separated into a reversible and an irreversible component, in the bottom diagram. We see that it takes some weeks before the maximum reversible strain is developed. There is a tendency for the

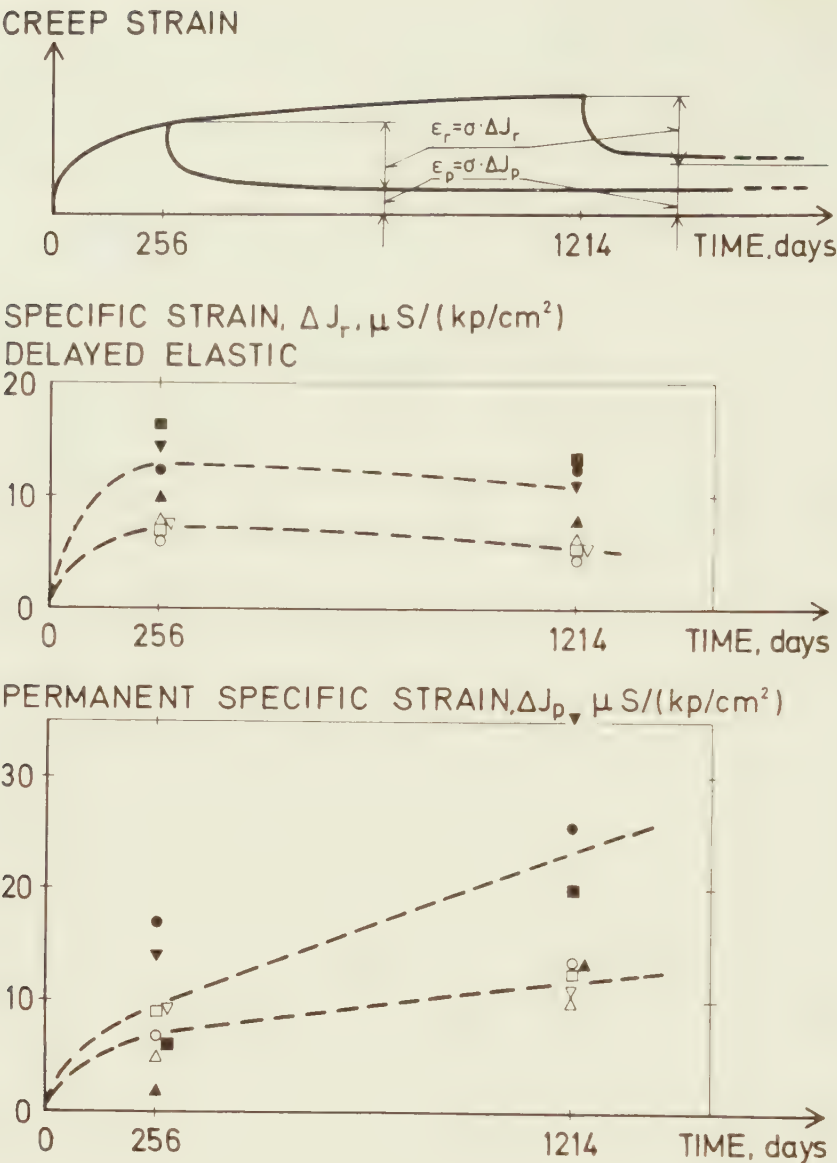


FIG. 6.9 The components of time dependent strain in cellular concrete. Top: Sketch of the strain development. Bottom: Evaluation of the delayed elastic and the permanent components. The dotted curves indicate the trends. Experiments described in Appendix C. Each point is the mean of two results.

Factory	K	B	S	D
Symbols: 43% RH	○	□	△	▽
Water saturated	●	■	▲	▼

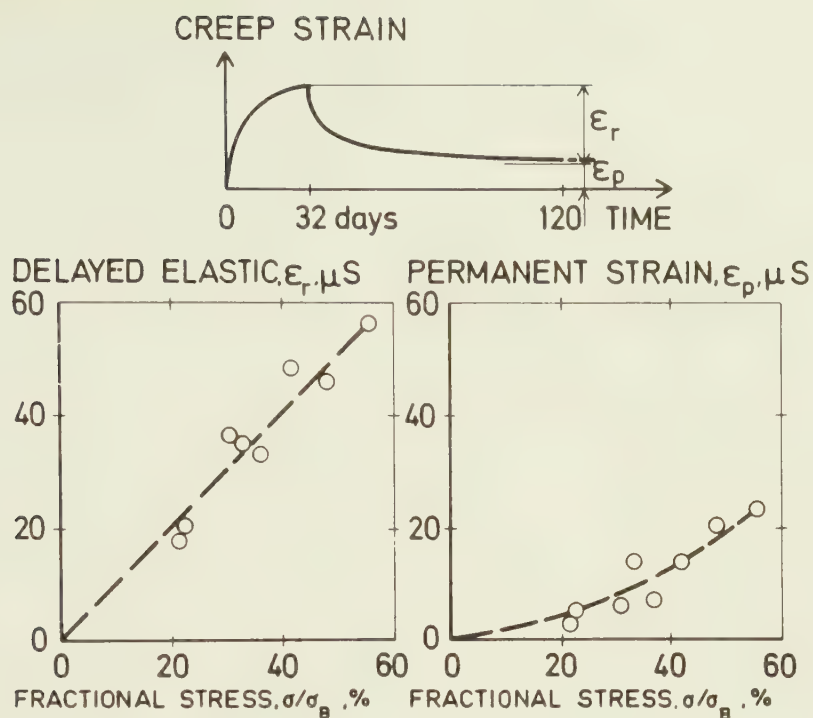


FIG. 6.10 Creep strain of cellular concrete separated into delayed elastic, ϵ_r , and permanent strain, ϵ_p . The dotted curves indicate the trends. Data from *Nielsen(1968b)*. (Dalby I Siporex).

Mean of flexural strength $\sigma_B = 9.6 \text{ kp/cm}^2$.
Mean of $E_0 = 15100 \text{ kp/cm}^2$.

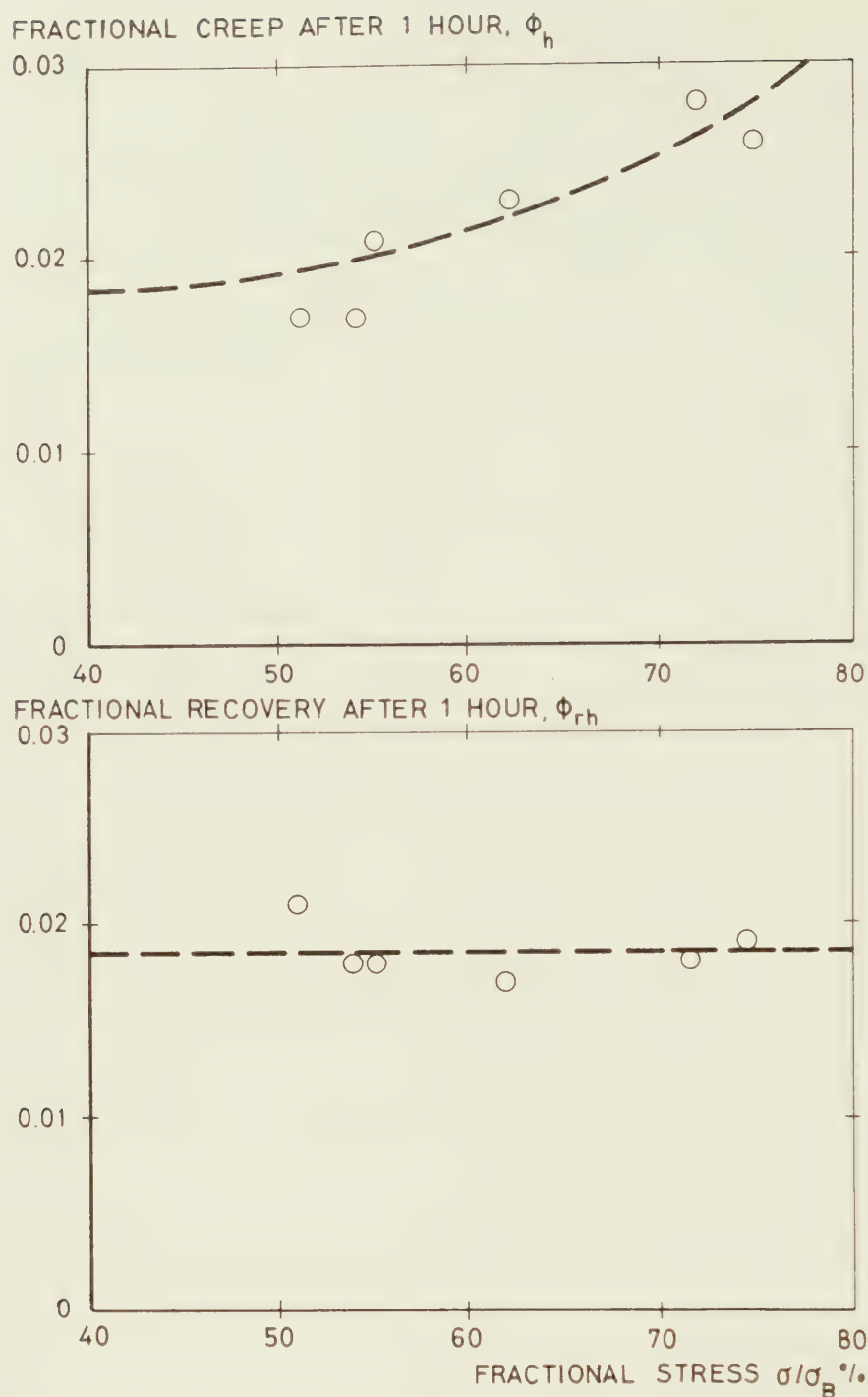


FIG. 6.11 Fractional creep and fractional recovery after 1 hour for laminated wooden beams as functions of fractional stress σ/σ_B . From *Littleford (1966)*. (ϕ_h is defined by: $\Phi = \phi_h \cdot t^b$, t in hours).

final values of the reversible component to decrease with time, certainly caused by continued hydration.

Results from my own investigation of four cellular concretes are shown in FIG. 6.9 (experimental description in Appendix C and in *Nielsen (1968b)*). FIG. 6.9 shows the results of the main experiment. Half of the beams were unloaded after 256 days and the other half after 1214 days under load. The results are similar to Illston's, FIG. 6.8. The final delayed elastic component decreases with time, probably because of hydration. The initial elastic strain also decreases with time, as shown in FIG. C.4.

*2 That the delayed elastic strain is linear with stress at all stress levels is supported by the following observations.

FIG. 6.10 shows the results of a pilot test on cellular concrete using higher stress levels than in my main experiment. The experimental method was the same. In spite of the great scatter, which is probably due to the sampling procedure, the reversible component appears to vary linearly with stress.

Bach (1972) also has demonstrated the linearity of the delayed elastic component with stress. He examined creep and recovery of wood in compression (specimen size 25 x 25 x 75 mm). The specimens were loaded for different times (max. 1 week) at stress levels of 10, 20, 30, ... and 90% of the crushing strength. The recovery showed linearity with stress at all stress levels.

The linearity of the delayed elastic strain is also implied by the results of *Littleford (1966)*, described below.

*3 The final value of the delayed elastic strain stands in a certain proportion to the instantaneous elastic strain at unloading. This is stated here more or less as a hypothesis. The examples given below do not refute this hypothesis. The hypothesis

TABLE 6.1. FINAL FRACTIONAL RECOVERY, ϕ_{re}

Material	Rel.hum. %	ϕ_{re}	Data evaluated from
Concrete, age 10 days	90	0.18 - 0.40	<i>Illston (1965)</i> , FIG. 6.8
Concrete, age 10 to 108 days	90	0.15 - 0.39	<i>Illston (1965)</i>
Concrete	~ 65	~ 0.33	Hummel, according to <i>L.F. Nielsen (1967)</i>
Cellular concrete	43	0.08 - 0.18	<i>Nielsen (1971b)</i>
Cellular concrete	60 - 80	0.08 - 0.18	<i>Schäffler (1960)</i>
Cellular concrete	Saturated	0.17 - 0.26	<i>Nielsen (1971b)</i>
Gluelaminated wooden beams	65 - 75	0.13 - 0.15	<i>Littleford (1966)</i>
Wooden beams, pine	65	0.12 - 0.14	<i>Cederberg and Danielsson (1970)</i>
Hardboard	32	0.2 - 0.3	<i>Lundgren (1968)</i>
Hardboard	65	0.5 - 0.6	<i>Lundgren (1968)</i>
Hardboard	90	1.0 - 1.4	<i>Lundgren (1968)</i>
Oak (perpendicular to fibers)	65	~ 0.65	<i>Ethington and Youngs (1965)</i>

is limited to a condition of climatic equilibrium. The final delayed elastic strain per unit stress can be thought of as a kind of materials constant governed by the same type of microstructural characteristics as Young's modulus. Using an analogy from electronics, it can be said that as a capacitor can hold a certain charge of electricity, so a specimen of the material can hold a certain "charge of elasticity".

Note that statements 2 and 3 imply a linear relationship between initial strain and stress.

Statement 3 refers to the instantaneous elastic strain at unloading instead of that at loading. This is convenient because the instantaneous elastic strain at unloading is easy to determine and it does not include any permanent part.

A further motivation to present the recovery data this way is that statement 3 seems to be valid if sorption has taken place during loading, provided that moisture equilibrium has been obtained before unloading. In this case it is obviously the instantaneous strain at unloading which is the proper reference.

To illustrate the validity of statement 3 a number of values of final fractional recovery, ϕ_{re} , are collected in TABLE 6.1. The relative humidities quoted in the table are the values with which the specimens are assumed to be in equilibrium at unloading.

The first values in the table are taken from the experiment by *Illston (1965)* described above. Illston does not himself establish a connection between the instantaneous and the delayed elastic component. I have used his data to calculate the values given in TABLE 6.1.

The ϕ_{re} values on cellular concrete at the 43% level found by me coincide with values for the 60 to 80% RH level evaluated from the experimental results of *Schäffler (1960)*.

Φ_{re} values on gluelaminated wood beams found by *Littleford* (1966) agree well with values I have calculated using data from the master thesis by *Cederberg and Danielsson* (1970).

Lundgren (1968) has studied the deformational behaviour of wood based materials. He has not made unloadings at different times for the same materials, so the hypothesis of a final value for the delayed elastic strain can not be tested using his results. However, he measured both creep and recovery after very long times. Hence, the final fractional recovery can be estimated. Such results for hardboard under different moisture conditions are shown in TABLE 6.1. Recovery curves are seen in FIG. 7.9.

Lundgren's data show that the final fractional recovery increases with increasing relative humidity. I found the same tendency in my experiments on cellular concrete.

*4 The delayed elastic strain takes a certain time to reach its final value. According to FIG. 6.8 and FIG. 6.9 this time is 50 to 100 days for concrete and 100 to 250 days for cellular concrete.

From FIGS. 5.8, 5.12, and 6.8 it is seen that the recovery is measurable for a longer time than the load has been applied. From FIG. 5.8 it is seen that a power function can be applied to the recovery curve until about 200 days, after which the recovery slows down, but it has not reached its final value even after 1000 days.

For loading times shorter than the time necessary to reach the final value of the delayed elastic strain this phenomena may be expected as a consequence of the Boltzmann superposition principle. But when the unloading is made after the final value has been reached, the recovery also continues a much longer time than that necessary to fully develop the final delayed elastic strain. Statement 4 is thus supported.

*5 The growth of the viscous strain with time is not linear as the term "viscous" implies. As seen from FIGS. 6.8 and 6.9 the viscous, or permanent, strain develops rather fast in the beginning, then slows down, but it never ceases to grow. Hence the never-ceasing deformation of most materials under load is due exclusively to the growth of the viscous strain component.

At short times the development of the viscous strain may be approximated by the tangent to the viscous strain curve at the origin. This means that at short times, for example during vibrations, the viscous strain can be considered to be linear with time and the behaviour of the material can be described in terms of linear viscoelasticity.

*6 The development of the permanent component also depends on the stress level. At low stress levels no permanent strain is developed during the first minutes." (This is shown for cement paste by *Sellebold (1969)*, for concrete by *Illston (1965)* and for wood-based sheet by *Lundgren (1968)*, among others.) At high stress levels permanent strain occurs also at short times as is seen in a conventional stress-strain experiment.

At high stress levels the viscous strain increases faster than at lower. The nonlinearity between stress and strain discussed in Section 6.1 is thus due to the nonlinearity of the viscous strain. This agrees with the assumption that the viscous strain component is caused by irreversible changes in the microstructure, for example dislocation motion or microcracking (*Müller (1969)*). These processes become intensified at higher stress levels. At sufficiently high stresses they lead to failure of the specimen, i.e. creep rupture (see FIG. 2.4).

The nonlinearity between permanent strain and stress is illustrated by FIG. 6.10 for cellular concrete.

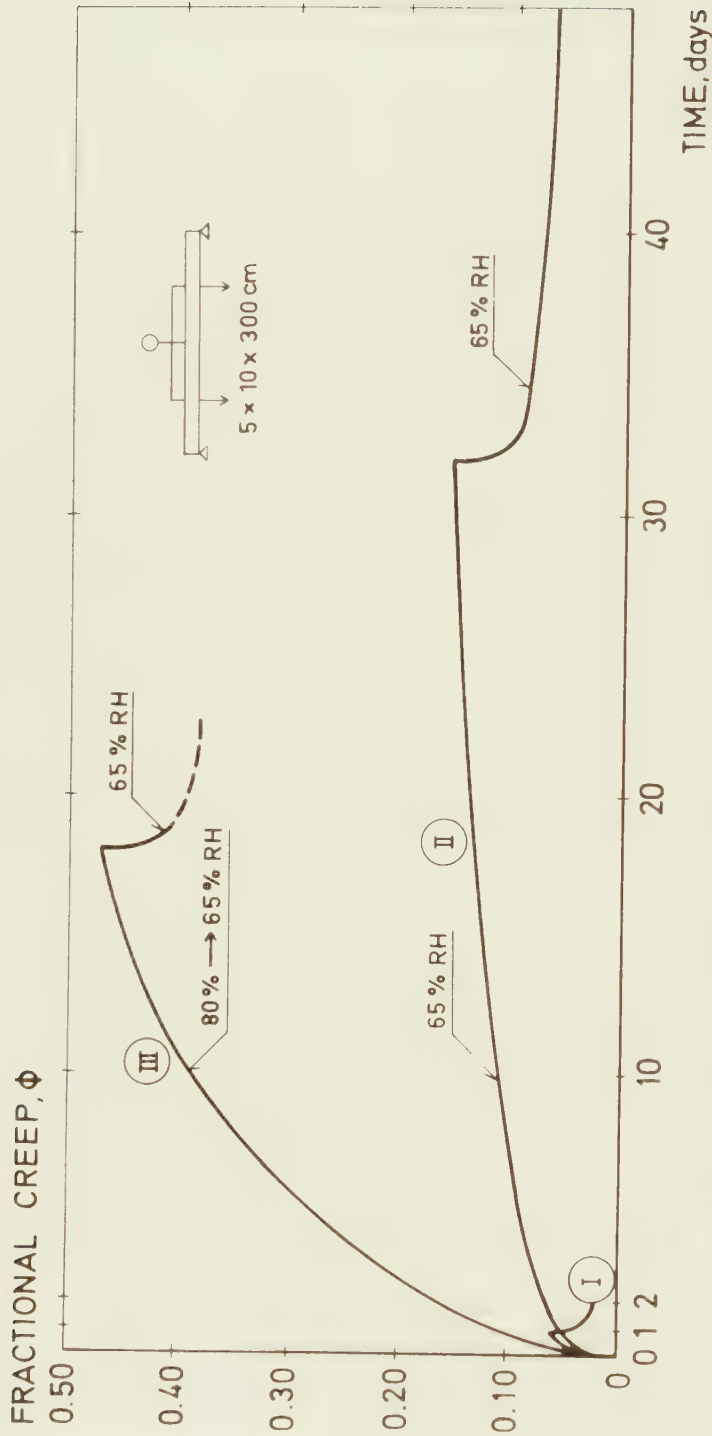


FIG. 6.12 Fractional creep and recovery of wood, Swedish pine, grade T 300. Curve I and II: Constant climate 65% RH. Curve III: Drying at 65% RH from equilibrium at 80% RH. The drying out is almost completed after 18 days, when the beam is unloaded. Creep is measured from 1/256 day. Data from *Cederberg et al. (1970)* and *Nielsson et al. (1970)* replotted here.

A further illustration is provided by *Littleford (1966)* for glue-laminated wood beams. He used stress levels of 51% to 75% of the bending strength. The fractional creep, Φ_c , obeyed a power function, $\Phi_c = \Phi_h \cdot t^b$.

Fractional recovery, Φ_r , also obeyed a power function during the first 2000 hours of recovery, $\Phi_r = \Phi_{rh} \cdot t^b$. In FIG. 6.11, Φ_h and Φ_{rh} are plotted vs. stress, σ/σ_B . Φ_h increases with stress in agreement with eq. (6.5) and FIG. 6.6. Φ_{rh} is constant, showing that the delayed elastic deformation varies linearly with stress.

The principles describing the stress and time dependence of the strain components have so far been discussed for constant climate conditions. The influence of climate variations shall now be considered (these problems are treated further in Chapter 7).

A variation in climate while the load is on, will usually cause an increase in the permanent strain component.

An example of this is shown in FIG. 6.12, which shows fractional creep and recovery of wooden beams, 5 x 10 x 300 cm. The experiments are described in Appendices A and B. Curve II shows creep and recovery at constant climate, 65% RH, 20°C. Curve III shows creep while drying from equilibrium at 80% RH to equilibrium at 65% RH, 20°C. The drying was essentially complete after 18 days. Beam III was now essentially in the same moisture condition as beam II. The recovery is seen to be of the same order of magnitude for the two beams, i.e. the large difference in deformation between the two beams is due to the permanent component. The same phenomenon is seen at unloading in my sorption creep experiments on cellular concrete, the creep parts of which are shown in FIG. 7.12 (*Nielsen (1968b)*).

A second example deals with sudden climate variations during the time under load or during the recovery.

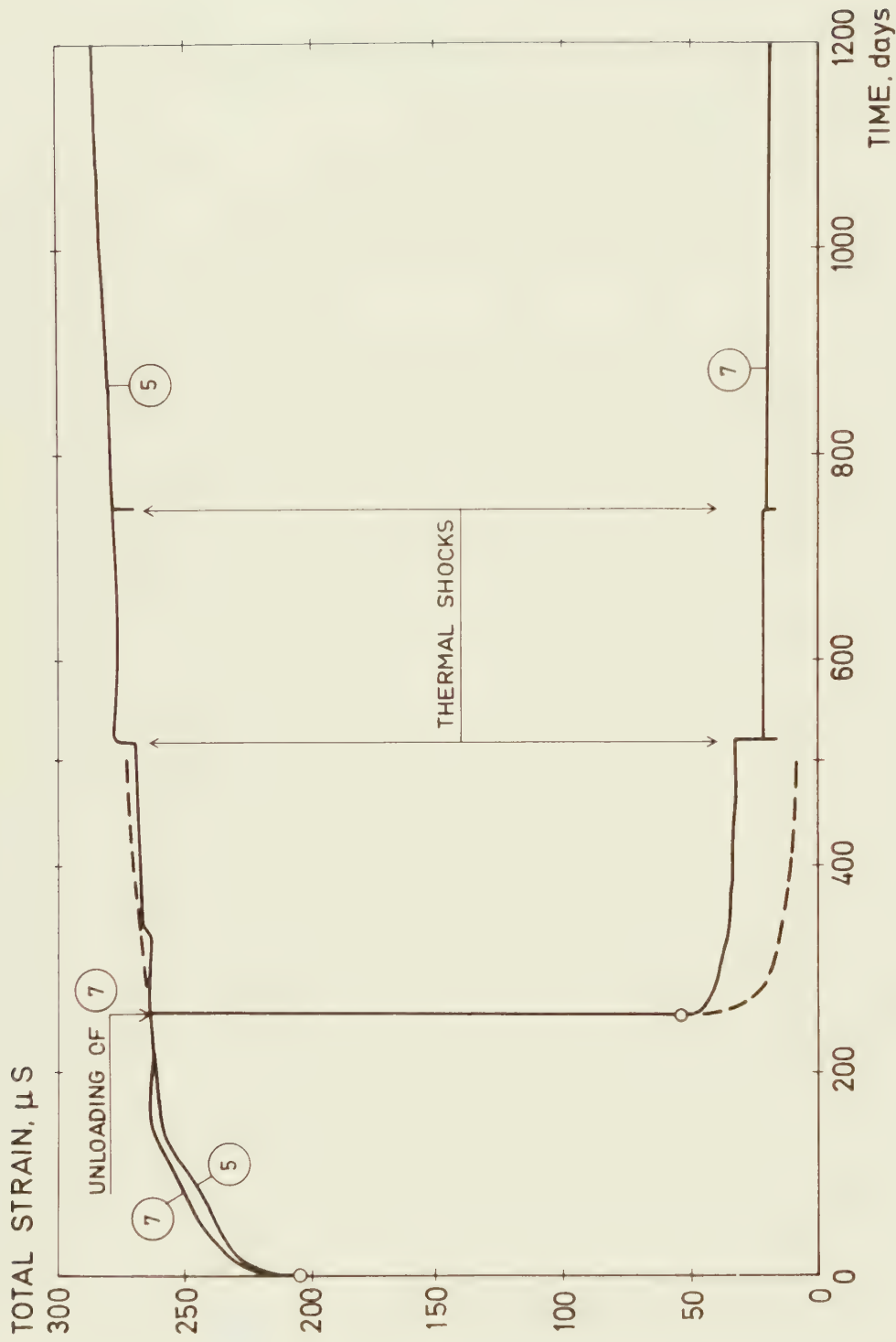


FIG. 6.13 Total strain curves for cellular concrete (Ytong, factory K, $\sigma = 4 \text{ kp/cm}^2$). The influence of thermal shocks is seen. The dotted line shows recovery predicted using the "virgin creep curve principle". (Specimens from the investigation described in appendix C.)

FIG. 6.13 shows total strain curves from my investigation of cellular concrete (*Nielsen (1968b)*). It is seen that the thermal shocks (sudden heating to 50-80°C, which were caused by trouble with the climate control apparatus) caused an increase in creep, but also increased recovery. An observation of increased recovery caused by climate variation is also reported by *Lundgren (1968)* for woodbased materials and by *Kingston and Armstrong (1951)* (see FIG. 7.21) for wooden beams. It has also often been observed in experiments made with no climate control. This influence on the recovery may partly be explained by the creation of internal stresses. *Müller (1969)* discusses this phenomenon.

None of the examples given above refute the statements 1 to 6. On the other hand the examples cannot be considered as sufficient evidence to establish the general validity of the statements. My hope is that the statements may provide a conceptual framework for thinking about these problems as well as for designing proper experiments.

6.3 The Boltzmann superposition principle

For practical design purposes there is a demand for rules to calculate deformations during stress variations. Some of these "superposition principles" will be discussed below.

Boltzmann (1876) has given a famous contribution to the theory of viscoelasticity. Boltzmann derived expressions for relaxation, creep and recovery after different loading periods and for oscillatory loading. He also showed the validity of his equations by ingeniously simple experiments on a glass thread in torsion.

Boltzmann's fundamental assumptions were the following:

1. Stress and strain are linearly related.
2. All strains are reversible.

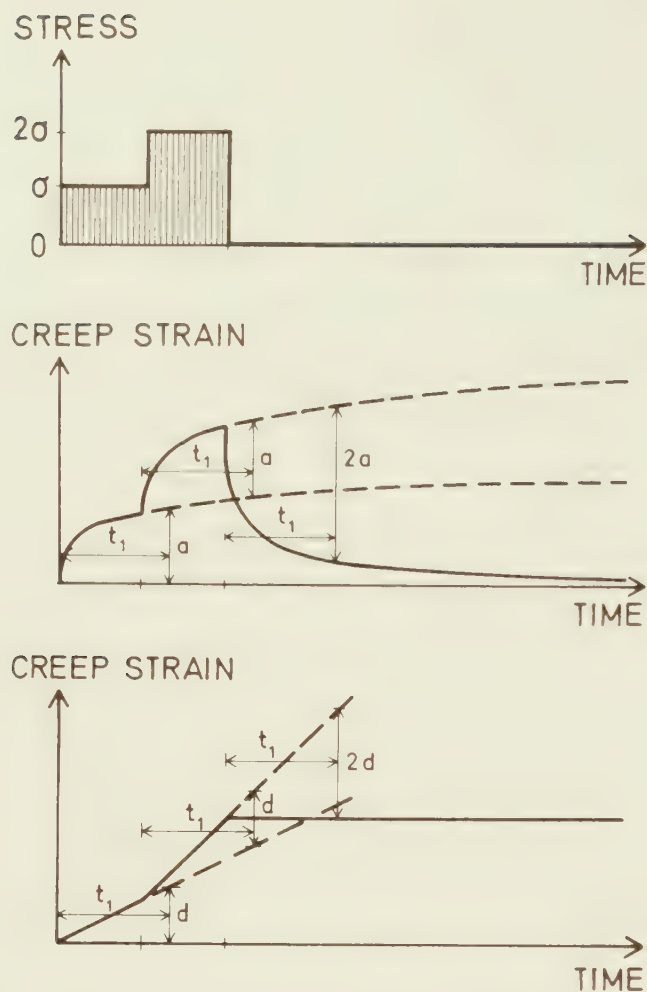


FIG. 6.14 The Boltzmann superposition principle applied to creep experiments. Top: Stress history. Middle: Material showing no irreversible strain. Bottom: Linear viscous material.

3. The strains are small.
4. The state of the material depends on its total stress-strain history, i.e. it has a "memory".

The *Boltzmann superposition principle* is a consequence of these assumption. When applied to creep, the principle stated, that the creep at a certain time can be found by superimposing the creep caused by all previous loadings, as shown in FIG. 6.14, center diagram. Unloadings are calculated as negative loads.

Boltzmann put great emphasis on assumption 2 in his discussion of the superposition principle, as well as in his experimental work. In his experiment, Boltzmann subjected a glass thread to mechanical conditioning by a number of vigorous twistings ($\sim 540^\circ$). He found extremely good agreement between the principle and the experimental results after this preconditioning treatment. Recovery after different long loading times was measured.

However, assumption 2 does not necessarily have to be obeyed. On the last page of his paper Boltzmann points out that in special cases his equations are transformed to the strain equation for a fluid. This means that the superposition principle is valid for linear viscous materials (see FIG. 6.14, bottom diagram) and for so-called strain hardening materials obeying for example eq. (6.7). (Strain hardening is defined in Section 6.4.) Thus, the Boltzmann superposition principle may be applied to materials showing both reversible and irreversible strain components. These components must first be separated, and then superposition applied to each individually.

The Boltzmann superposition principle is treated in all textbooks concerning rheology of solids, but few experiments demonstrate its validity. Leadermann points out (in his discussion of the paper by McHenry (1943)), that the principle is valid for polymers when permanent deformations do not occur. *Sellebold*

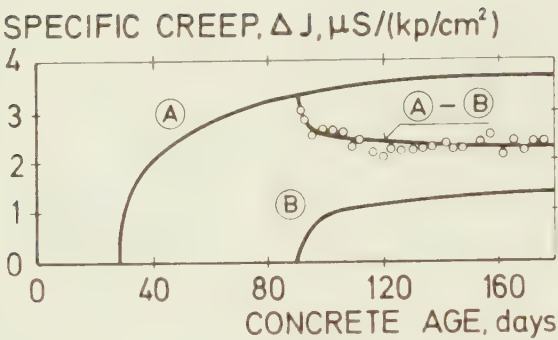


FIG. 6.15 The principle of superposition applied to concrete by *McHenry (1943)*. A and B are experimental creep curves. (A-B) is the calculated recovery. The circles represent experimental recovery values.

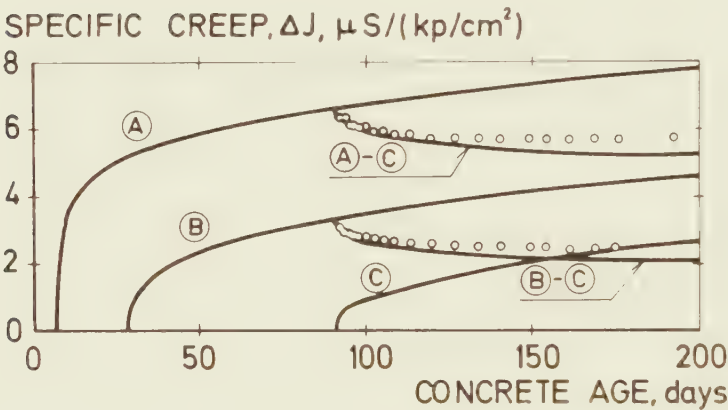


FIG. 6.16 Creep and creep recovery from *Bäckström (1956)*. (A-C) and (B-C) are calculated recovery curves. The circles show measured recovery values.

(1969) has recently shown its applicability to hardened cement paste at low stress levels and for short loading times.

A common misunderstanding of the principle is to apply it to the virgin creep curve without any separation of the creep strains into reversible and irreversible parts. Since the viscous strain varies non-linearly with time, as shown in FIGS. 6.8 and 6.9, one will find too much recovery using this "virgin creep curve principle". A graphical demonstration of this is shown by the dotted line in FIG. 6.13. The non-applicability of the virgin creep curve principle to building materials can also be demonstrated mathematically.

In Chapter 5.3 it was pointed out that virgin creep of most building materials can be described by the power function eq. (5.2) with $b < 1$. According to the virgin creep curve principle an unloading curve can be calculated in the following way (unloading at $t = t_1$):

$$\epsilon_c = At^b - A(t-t_1)^b$$

It can be shown that ϵ_c calculated according to this equation will approach zero, when $b < 1$, i.e.

$$\begin{aligned} \epsilon_c &\rightarrow 0 \text{ for} \\ t &\rightarrow \infty. \end{aligned}$$

This is not true for most building materials, which show some viscous strain. Therefore this virgin creep curve principle can not be applied. But it can be used as an approximation when the viscous strain is without importance.

The experiments by *McHenry (1943)* on concrete are often quoted as proof of the Boltzmann superposition principle. However, McHenry proposed a modification of the "virgin creep curve principle", mentioned above. He took into account the effect of the concrete age at loading. His proposal is illustrated by

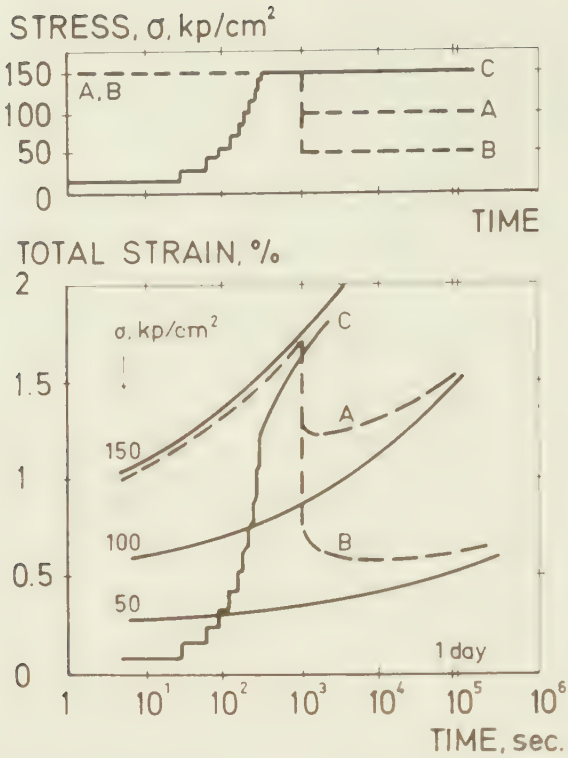


FIG. 6.17 Experiments on polypropylene showing total strain curves at three constant (tensile?) stresses, σ , and the strain development when stresses are changed in different ways A, B and C. From *Turner (1965)*.

FIG. 6.15, which is taken from his original paper. The concrete has the virgin creep curve A when loaded 28 days old, and the virgin creep curve B when loaded 90 days old. McHenry postulates that the recovery curve of concrete A upon unloading at 90 days will equal A-B as shown.

According to my discussion above, the unloading curve in FIG. 6.15 ought to lie higher, as demonstrated for example in the similar experiments by *Bäckström (1956)* (FIG. 6.16). The agreement between the experimental curve and the predicted curve A-B in FIG. 6.15 is good. But the irregular shape of the experimental curve indicates that some variations either in temperature or in humidity have occurred during this experiment, causing further recovery (confer also FIG. 6.13). In a footnote to his paper McHenry admits that "... the average agreement was somewhat less satisfactory than that shown by the figure, indicating ... that 100 per cent precision should not be assigned to the superposition principle."

McHenry views his method as a working approximation to the truth and as such it has won wide acceptance.

Lundgren (1968) used the "virgin creep curve principle" on basic creep experiments with hardboard. His results also show that the use of this principle overestimates the recovery, i.e. underestimates the permanent strain.

Turner (1965), an expert on polymers, states that the Boltzmann superposition principle is not valid for real materials because of their nonlinearity with stress. He proposes the graphical method illustrated in FIG. 6.17 for polymers.

The shortcoming of the virgin creep curve principle is even more pronounced when sorption creep is considered.

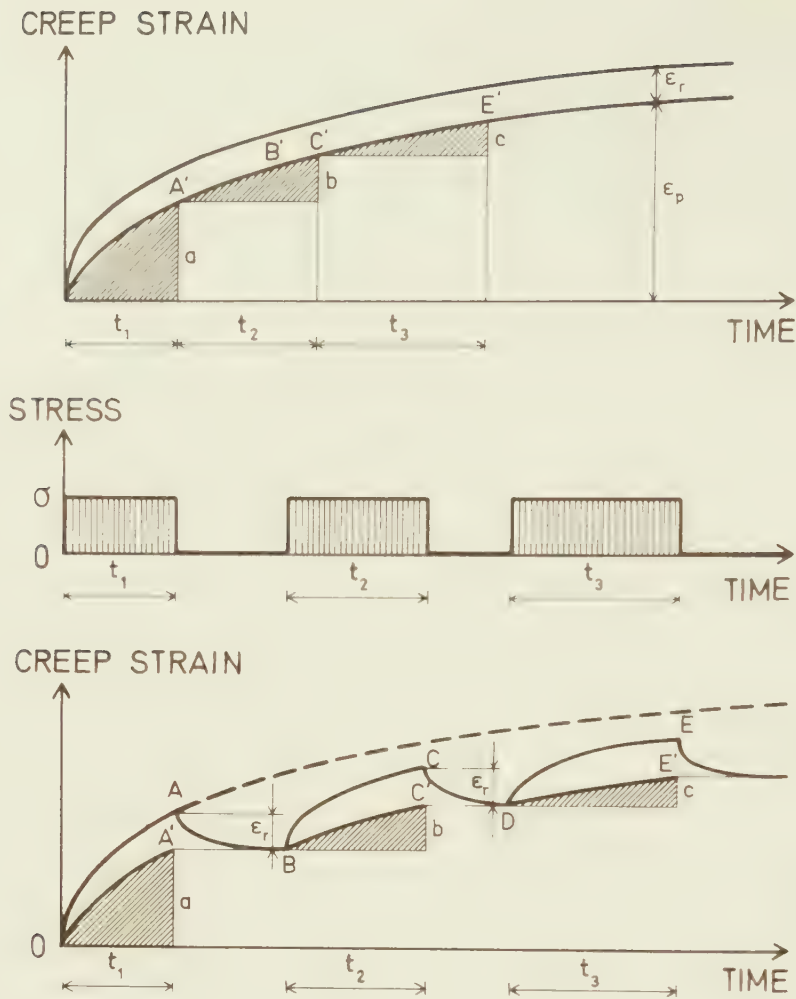


FIG. 6.18 The development of creep under intermittent loading. Top: Virgin creep curve separated into reversible and irreversible components. Middle: Stress variations. Bottom: Strain response.

6.4 Graphical superposition

This section deals with a graphical construction of creep curves during stepwise changing load, based on the principles stated in Section 6.2. The construction can be seen as an illustration of the Boltzmann superposition principle. But it may also be made when the stress-strain relationship is nonlinear. Loading-unloading cycles and load variations between different stresses are treated. The phenomenon of strain hardening is described.

According to the principles stated in section 6.2, the development of the different creep strain components during periods of intermittent loading and unloading can be assumed to be as illustrated in FIG. 6.18. The loading times are assumed to be sufficiently long for the delayed elastic strain to fully develop. During the first loading period a great portion, a , of the viscous strain is developed. During the unloading period only the reversible strain, ϵ_r , is recovered. During the next loading period the viscous strain is developed further from the point A' on the permanent part of the virgin curve. It increases with the quantity b . (Physically this may be thought of as a continuation of the minute ruptures which were started during the first loading period.) During the next unloading period the delayed elastic component, ϵ_r , is once more recovered. The next loading period gives the irreversible strain, c , and so on. The reloading curves, BC and DE , do not have the same shape as the virgin creep curve OA , because the irrecoverable parts are not the same. This is the reason why it is necessary to distinguish between the virgin creep and the reloading curves, as pointed out in FIG. 2.3.

A consequence of this approach is that if all the unloading periods are excluded, i.e. point B is moved to point A' , point D to C' etc., then the points C and E will lie on the virgin curve. *Lundgren (1957)*

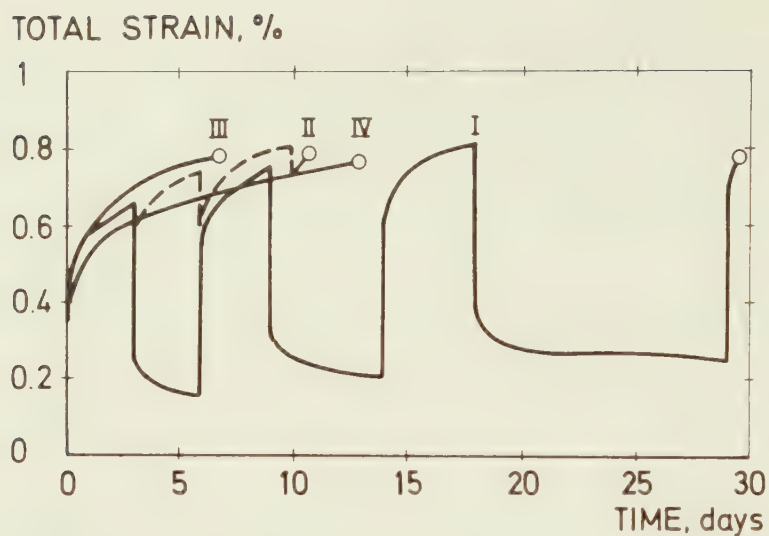


FIG. 6.19 Total strain curves from a creep experiment on hardboard in tension (*Lundgren (1957)*).

- I. Periodic loading.
- II. Same as I, but unloading periods excluded.
- III. and IV. Virgin creep curves for the same material.

made experiments on hardboard, which illustrate this point (FIG. 6.19). Many other authors have reported experiments where increase of creep during periodic loading was observed. But it is seldom that the basic virgin creep curve is also given, as in Lundgren's experiment. *Neville (1970a)* reports experiments on concrete, *Schäffler (1960)* treats cellular concrete, *Kingston and Clarke (1951)* deal with wood, and *Norén (1968)* has examined nailed joints.

In connection with intermittent loading the following observation for concrete, made by *Bazant (1968)*, shall be mentioned. For shorter load-unload periods between two stress levels, for example due to traffic, the resultant creep curve may lie over the virgin creep curve determined at the highest of the two stress levels. The reason for this may be fatigue or that heat is developed during the vibrations, so that the material creeps at a higher temperature.

The intermittent loading experiment in FIG. 6.18 draws our attention to the intermittent loading in a conventional stress-strain experiment. A permanent strain remains when a material is unloaded from a high stress level in such an experiment. The material is said to have "strain hardened". The physical processes causing the permanent component of the creep strain may be assumed to be the same as the processes causing permanent strain in the stress-strain experiment. Therefore, we may also use the term *strain hardening* about the "mechanical conditioning" which happens during creep.

When a material is strain hardened by loading it to a rather high stress level, it is difficult or impossible to detect any viscous strain upon reloading to a much lower stress level. It is important to be aware of this when considering the superposition principles. As mentioned above, Boltzmann strain hardened his glass threads before testing his superposition principle.

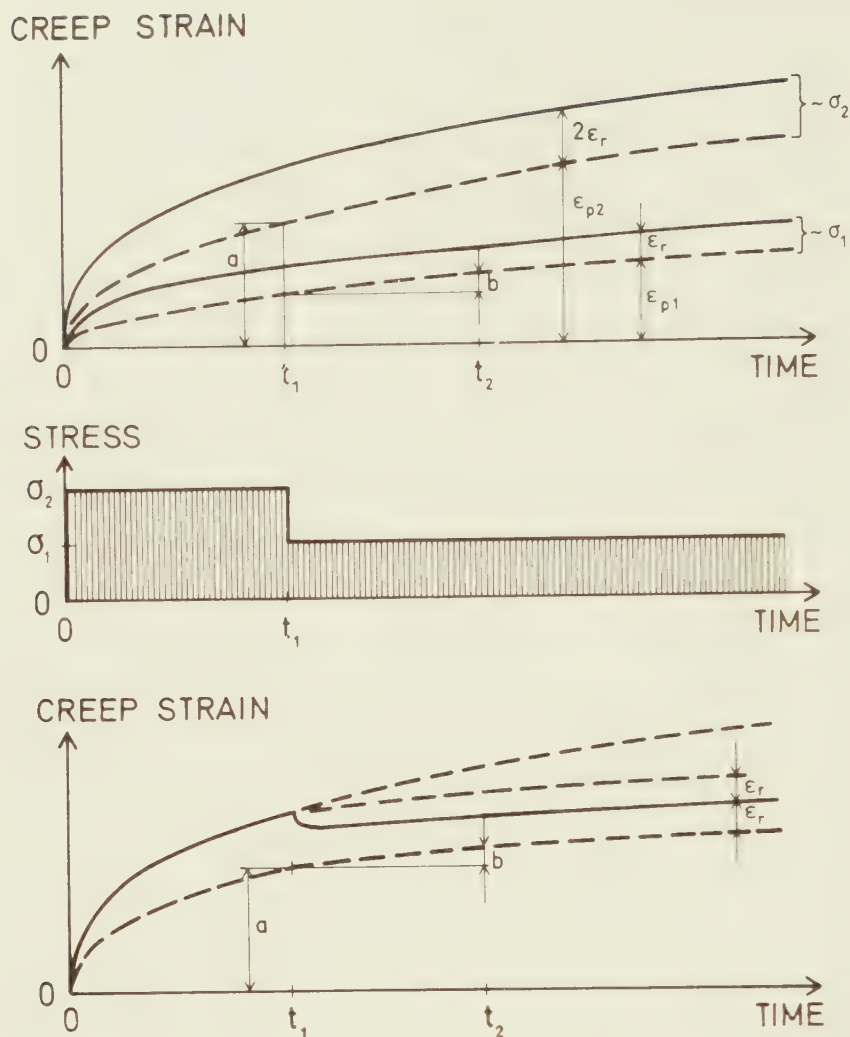


FIG. 6.20 A proposal for a graphical method to construct a creep curve during a step loading history. Top: The virgin creep strains at σ_1 and σ_2 separated into a reversible, ϵ_r , and an irreversible component, ϵ_p . Middle: The stress variation. Bottom: The resultant creep curve.

The case where the load varies between different stress levels (of the same sign) shall now be treated briefly. An attempt at a graphical construction is shown in FIG. 6.20. In this construction it is assumed that the material after unloading at time t_1 from σ_2 to σ_1 , will follow the virgin curve at stress σ_1 . But since the material already has a viscous deformation of the magnitude a , i.e. it has strain hardened, it is not certain that the stress σ_1 is sufficiently high to continue to activate the microstructural processes which give rise to the viscous strain. Whether the viscous strain after the time t_2 is b or less will depend upon the type of material, and it can only be determined by direct measurements.

Turner (1965) has considered the nonlinear superposition problem for polymers. He shows results from experiments on polypropylene (FIG. 6.17). The virgin curves for three stress levels and the strain development when the stress is changed in three different ways (A, B, C) are shown. From such experiments and on the basis of his wide experience Turner concluded that the resultant creep curves will asymptotically approach the virgin curve for the final strain. Turner's statement is important and will probably work well for materials which exhibit little permanent strain. However, when appreciable viscous strain is developed it is clear that this principle can not hold.

6.5 Mathematical superposition

To establish a generally applicable superposition principle requires that the two time dependent strain components can be expressed as functions of time, stress and stress history (confer Section 6.3). This is difficult and so far no general theory has been put forward. In this Section I will describe one of the approximate methods available.

This method has been proposed by *L.F. Nielsen (1967,*

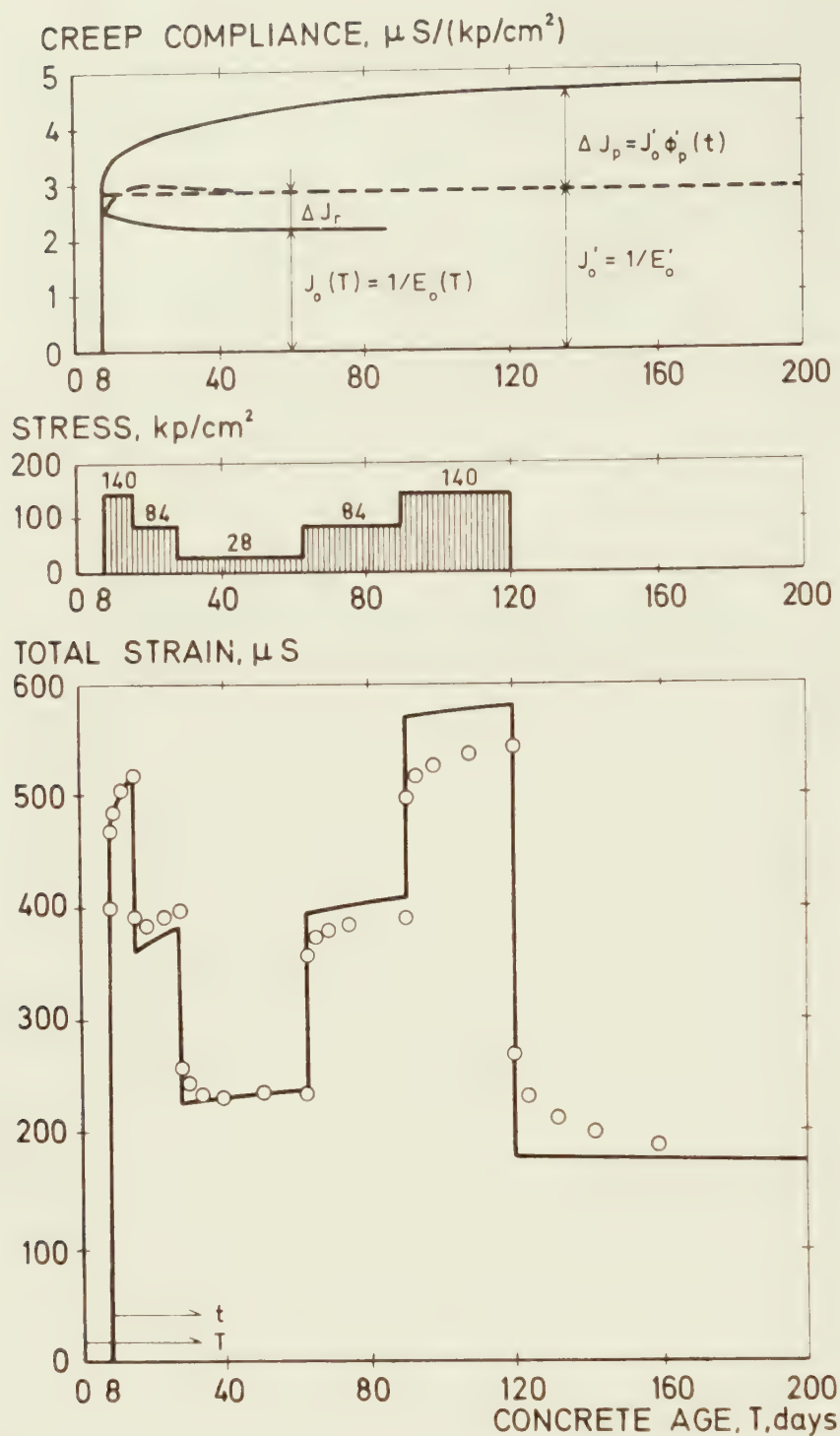


FIG. 6.21 Superposition using the modified Dischinger equation. Experimental data on concrete from Ross treated by *L.F. Nielsen (1968)*. Top: Virgin creep compliance separated into reversible and irreversible components. Middle: Stress history. Bottom: Total strain variation. Solid line = predicted curve. Circles = experimental points.

1968) to apply to concrete. It is based on statements 1 and 3 in Section 6.2 that the delayed elastic strain has a final value, which is a constant fraction of the instantaneous strain. These assumptions together with the fact that reversible strain is completed at a time which is short compared to the lifetime of a building led him to introduce a fictive modulus of elasticity E'_0 , which takes into account initial as well as delayed elastic strain.

His method of course does not apply while the reversible strain develops, but at longer times the method works. L.F. Nielsen is well aware of this limitation, but points out that considerable mathematical difficulties arise when a more exact calculation of the delayed elastic component is attempted. It is therefore not justified to make more exact calculations, particularly when one also considers the uncertainty in the parameters of the material.

L.F. Nielsen's method can be expressed mathematically in the following way:

The total strain is

$$\varepsilon_{\text{tot}}(t) = \varepsilon_0 + \varepsilon_r(t) + \varepsilon_p(t)$$

By dividing with the stress one obtains the compliance

$$J_1(t) = J_0 + \Delta J_r(t) + \Delta J_p(t)$$

The reversible components are combined into a fictive, time independent value $J'_0 = 1/E'_0$ (see FIG. 6.21, top):

$$J_1(t) = J'_0 + \Delta J_p(t)$$

Introducing a fictive, fractional, permanent creep,

$$\phi'_p(t) = \Delta J_p(t)/J'_0, \text{ gives}$$

$$J_1(t) = J'_0(1 + \phi'_p(t))$$

The contribution to the fictive, fractional, permanent creep of a stress step $\Delta\sigma_1$ applied at the time t_1

may be expressed as

$$\Psi(t) = \Phi'_p(t) - \Phi'_p(t_i) \quad (6.7)$$

t is measured from the application of the first load. This means that the irreversible strain component due to $\Delta\sigma_i$ increases from the point on the time function curve corresponding to the time t_i .

For a stress history with N steps one then gets the total strain

$$\varepsilon_{\text{tot}}(t) = \sum_{i=1}^N J'_O (1 + \Phi'_p(t) - \Phi'_p(t_i)) \Delta\sigma_i \quad (6.8)$$

where $\Delta\sigma_i$ is the increase of stress at the time t_i for the i 'th step.

L.F. Nielsen applied eq. (6.8) to experimental data by Ross. FIG. 6.21 shows one of the results. The values for J'_O and $\Phi'_p(t)$ are shown at the top of the figure. The stress history is shown in the middle, and the bottom part of the figure shows the predicted and the actual strains. The agreement between the two is sufficiently good for many design purposes.

L.F. Nielsen discusses the choice of the constant $J'_O (= 1/E'_O)$ and the time function $\Phi'_p(t)$, both of which depend on the quality and age of the concrete. $\Phi'_p(t)$ is evaluated on an empirical basis for basic as well as sorption creep.

He points out that the differential form of eq. (6.8) may be written ($\Phi'_p(t)$ replaced by $\omega(t)$).

$$\frac{d\varepsilon}{dt} = J'_O \left(\frac{d\sigma}{dt} + \sigma \frac{d\omega}{dt} \right) \quad (6.9)$$

Eq. (6.9) is a modification of the classical Dischinger equation:

$$\frac{d\varepsilon}{dt} = \frac{1}{E_O(t)} \frac{d\sigma}{dt} + \frac{\sigma}{E_O(0)} \frac{d\Phi}{dt}, \quad (6.10)$$

where $E_O(t)$ is the age dependent Young's modulus of concrete and $E_O(0)$ is its value at the start of the

creep experiment. Eq. (6.10) describes all creep as permanent, whereas eq. (6.9) has the advantage of taking the reversible creep into account in the fictive initial compliance J'_0 .

Eq. (6.9) can be transformed by dividing through with $\frac{d\omega}{dt}$. Hereby one obtains (since $J'_0 = 1/E'_0$)

$$\frac{d\epsilon}{d\omega} = \frac{1}{E'_0} \left(\frac{d\sigma}{d\omega} + \sigma \right) \quad (6.11)$$

This is the differential equation for a Maxwell body (see Section 5.4). This transformation corresponds to a formal change of the time scale. It can be used with advantage in structural design calculations (*L.F. Nielsen (1967)*).

The method described above is developed for concrete only. From Section 6.2 it is seen that the time dependent elastic strain also for other building materials seems to be a constant fraction of the initial recovery. If this hypothesis can be shown to have general validity, then the above mentioned method of calculation may be applied also to other materials. This method should also make it possible to adapt superposition to sorption creep, if the proper timefunction $\Phi'_p(t)$ can be found. Furthermore, it is possible that the nonlinearity between stress and creep strain described in Section 6.1 also could be incorporated. These possibilities ought to be further examined.

7. INFLUENCE OF CLIMATE

The climatic conditions in and around a specimen, i.e. its temperature and humidity conditions, influence the deformational behaviour greatly. Temperature influences the behaviour of all materials, whether hygroscopic or unhygroscopic. The influence of humidity is a special problem for the hygroscopic materials. The influence of temperature, when the material is in equilibrium at a constant temperature is treated in Section 7.2. Creep under constant moisture conditions is discussed in Section 7.3. In practice, however, the temperature as well as the moisture condition of a structure changes continuously during its lifetime. The influence of varying climatic conditions is treated in Sections 7.4 to 7.8.

7.1 Constant climate in general

Most creep experiments are carried out with the specimens in a condition of constant climate. (This statement does not include concrete, see Section 7.6). The specimen is brought into equilibrium with a certain temperature and relative humidity before loading. This does not imply that the temperature and moisture content inside the specimen are constant during the experiment. It is well known that deformation is accompanied by changes in temperature (*Müller (1969)*). Changes in moisture content may also occur as discussed by *Barkas (1949)*, and more recently by *T.C. Powers*.

In investigations of the "engineering" properties of materials internal moisture and temperature changes are not studied; only external deformations are measured. It is common to seal test specimens after they have been brought to the desired initial condition. It is then assumed that the moisture content of the specimen remains constant. However, during loading the state of moisture equilibrium will

change under the sealing, so that one does not know which climatic condition actually prevails. (Another complication with the use of sealings is to get them tight enough. I experienced some difficulties during my experiments on cellular concrete, *Nielsen (1968b)*).

A method to obtain well defined experimental conditions is to let conditioned air flow around the specimens, as described by *Wittmann (1970)* for small specimens of cement paste, and used by *Lundgren (1968)* for woodbased materials. In this case the specimens are not sealed. This method was also used in the experiment described in Appendix A.

It is occasionally discussed whether basic creep should be referred to the relative humidity of the environment or to the moisture content of the material. It is most convenient for the practical use of the data to describe creep as a function of relative humidity since it is this parameter which is known in practice. If moisture content is used as a parameter, one must know the sorption isotherm for the material in order to get the corresponding relative humidity.

In wood research it is most common to refer to the moisture content of the specimen. Because of sorption hysteresis the moisture content does not uniquely define the relative humidity, but can correspond to a range in the relative humidity. For wood this range may be 10% ($\Delta RH \sim 10\%$), as is seen from the sorption curves in FIG. B.10.

7.2 Creep at constant temperature

Isothermal creep increases with increasing temperature. An estimate of the order of magnitude of this effect for concrete and wood will be given.

The creep behaviour of concrete at elevated temperatures is rather complex. The most important reasons are probably that heating of concrete accelerates hydration and produces structural changes in the al-

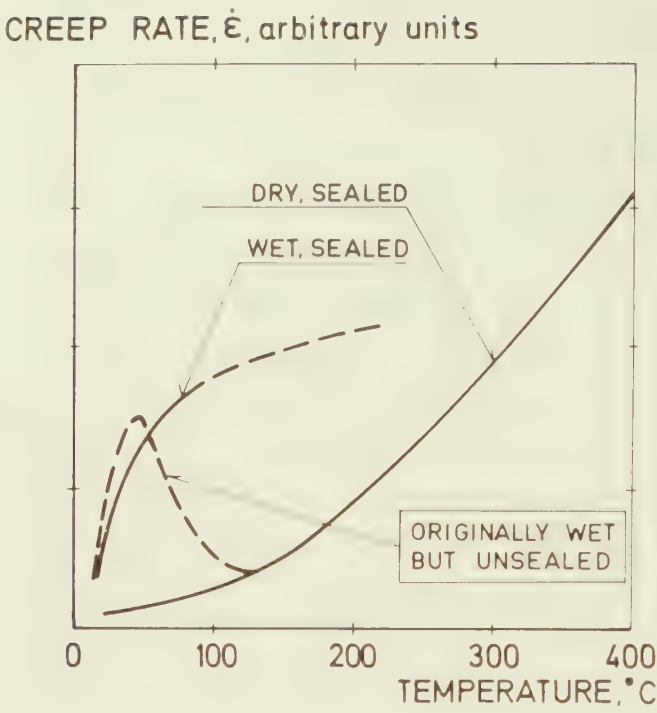


FIG. 7.1 Creep rate vs. temperature for concrete. From *Marechal (1970)*. The heavy lines show the creep rate at constant temperature for dry and wet materials. The dotted line shows the creep rate of an originally wet, unsealed specimen, corrected for shrinkage.

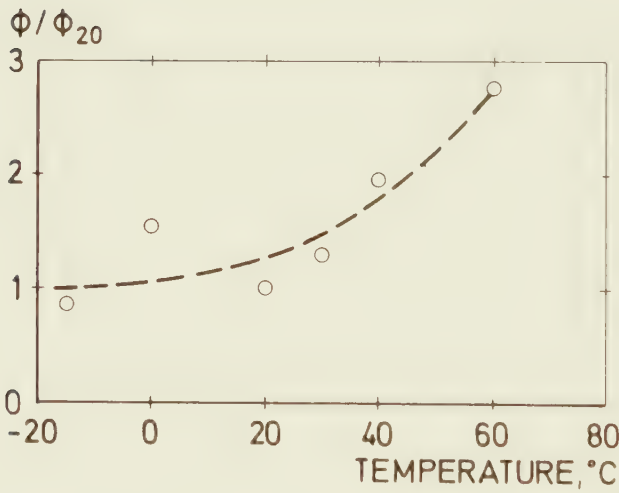


FIG. 7.2 Fractional creep of watersaturated sealed cement mortar beams after 24 days at different temperatures relative to that at 20°C. Raw data from *Hansen (1960)*.

ready hydrated material (*Sellebold (1969)*). Thus, the entire preconditioning treatment as well as the testing temperature influence the creep behaviour of concrete.

The examples given below are not intended to illustrate the very complicated relationships mentioned above; but rather to show that on an empirical level the influence of temperature on basic creep of concrete is similar to that found for other materials.

Marechal (1970) has studied creep of concrete under different moisture conditions in the temperature range from 20°C to 400°C . A graph from his investigation is given in FIG. 7.1, which shows creep rate a short time after loading versus temperature. Basic creep rates for dry and wet specimens are shown. The wet specimens creep much faster than the dry ones. The two curves have dissimilar shapes, but both creep rates increase with temperature, as is also found for other materials. The strain rate curve for the wet concrete has an opposite curvature to that for the dry material, which demonstrates the complicated creep behaviour of concrete.

FIG. 7.1 also shows the creep rate of originally wet specimens brought to equilibrium at the wanted temperature and then unsealed and loaded. In this case desorption and therefore shrinkage takes place upon unsealing. The curve in FIG. 7.1 is corrected for shrinkage rate according to eq. (7.1). The creep rate first follows the curve for the wet, sealed specimen. At higher temperatures the specimens dry very fast and therefore the curve for the strain rate changes direction and approaches that for the dry sealed material.

Hansen (1960) examined the creep of watersaturated cement mortar beams for 24 days after loading. FIG. 7.2 shows his results. The graph is constructed by dividing the fractional creep at 24 days at different temperatures by the fractional creep at 24 days and

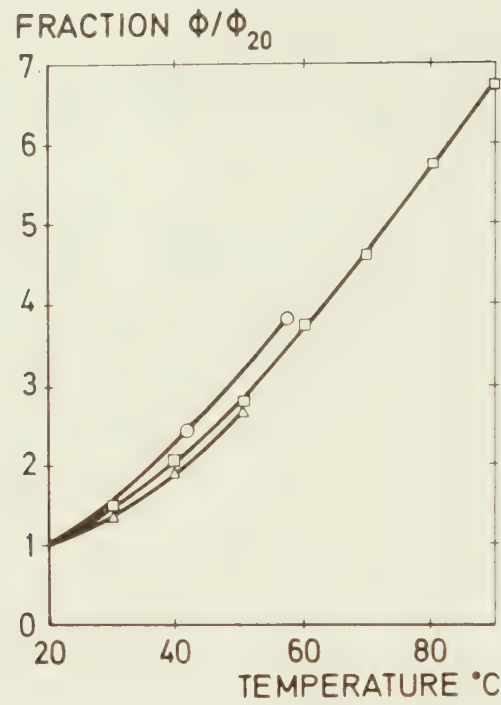


FIG. 7.3 Fractional creep of wood vs. temperature. The fractional creep at 20°C, ϕ_{20} , is unity. Experimental data from *Becker and Reiter (1970)* $\square$, *Kingston and Clarke (1961)*, $\circ$, and *Kitahara and Yukawa (1964)* Δ . Data treatment by *Wallin (1971)*.

20°C. This ratio increases about 200% from 20°C to 50°C. A similar increase is seen for the wet specimens in FIG. 7.1. - It should be pointed out that the temperature has another influence on the creep than on the initial deformation. This is seen from the fact that there is an increase in the fractional creep in FIG. 7.2. Hansen found the decrease of E_0 from 20°C to 60°C to be only 25%.

For wood several results are available. *Becker and Reiter (1970)* made stress relaxation measurements in flexure at different moisture contents and temperatures. *Kingston and Clarke (1961)* and *Kitahara and Yukawa (1964)* made flexural creep experiments. J.Å. Wallin and I compared the results from these three papers (*Wallin 1971*). We transformed the relaxation values of *Becker and Reiter (1970)* to fractional creep values by means of eq. (D.4.). The comparison is shown in FIG. 7.3, in the form of fractional creep after one day at temperature T, Φ_T , divided by the fractional creep after one day at 20°C, Φ_{20} . The agreement between the results from these independent investigations is remarkable. The original experiments were carried out at different moisture conditions, but this apparently has no significant influence on the temperature dependence.

It is known that for wood, Young's modulus, E_0 , decreases about 10% when the temperature increases from 20°C to 60°C (*Thunell (1941)*).

Sauer and Haygreen (1968) tested hardboard in flexure, and found that at a stress intensity of 30%, an increase in temperature from 22.2°C to 33.3°C gave an increase in creep deflection of about 50%. The increase ranged from about 75% at low moisture contents ($u = 5\% \sim 20\% \text{ RH}$) to about 25% for board tested while absorbing.

Ogorkiewicz (1970) shows isometric stress vs. time curves and tensile creep modulus vs. time curves for polymers at 20°C and 60°C. An example of his curves

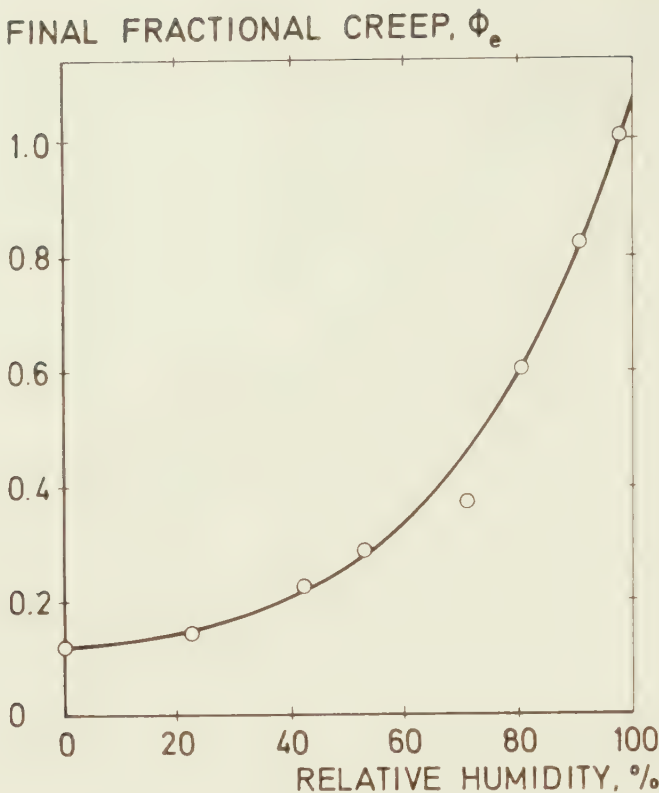


FIG. 7.4 Final fractional creep vs. equilibrium relative humidity for cement paste, ($w/c = 0.40$, compression test, stress intensity $\sigma/\sigma_B \approx 20\%$ Wittmann (1970).

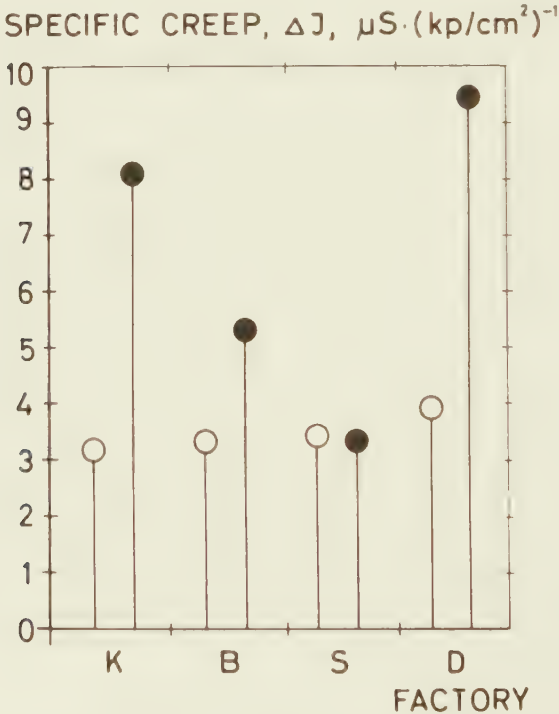


FIG. 7.5 Specific creep of cellular concrete after 24 hours. Samples from 4 factories tested in equilibrium with 43% RH, 0, and water-saturated, ●. (See Appendix C)

is given in FIG. 4.4.

To summarize: The effect of elevated temperatures is to increase isothermal creep. Creep at 50°C is at least 100% larger than that at 20°C for the experiments discussed here.

7.3 Creep under constant moisture conditions

Creep increases with increasing moisture content in the material. This will be illustrated by some experimental results.

Wittmann (1970) examined creep of cement paste as a function of relative humidity. A result from his investigation is shown in FIG. 7.4. The final fractional creep (calculated when the results are fitted to Ross' hyperbolic formula No. V in TABLE 5.2) is plotted vs. RH. A very marked increase is seen at the higher relative humidities. Wittmann proposes a physical explanation for this increase.

I have studied basic creep of cellular concrete at two different moisture conditions, 43% RH and water-saturation. The experiment is described in Appendix C and by *Nielsen (1968b)*. FIG. 7.5 shows specific creep after 1 day at the two moisture contents for beams from four factories. The agreement at 43% RH is fairly good. In the watersaturated condition, however, the results vary widely. The samples from factories K, B and D show from 50% to 100% more creep than at 43% RH, while those from factory S show no difference. This is remarkable and so far unexplained.

The influence of moisture on the creep of wood was also treated by *Wallin (1971)*. Basic creep results from 35 experiments by 10 authors were collected, including the results given in Appendix A. Corrections for stress and temperature dependence were made, and fractional creep after 1 year found by extrapolation, using power functions. These fractional

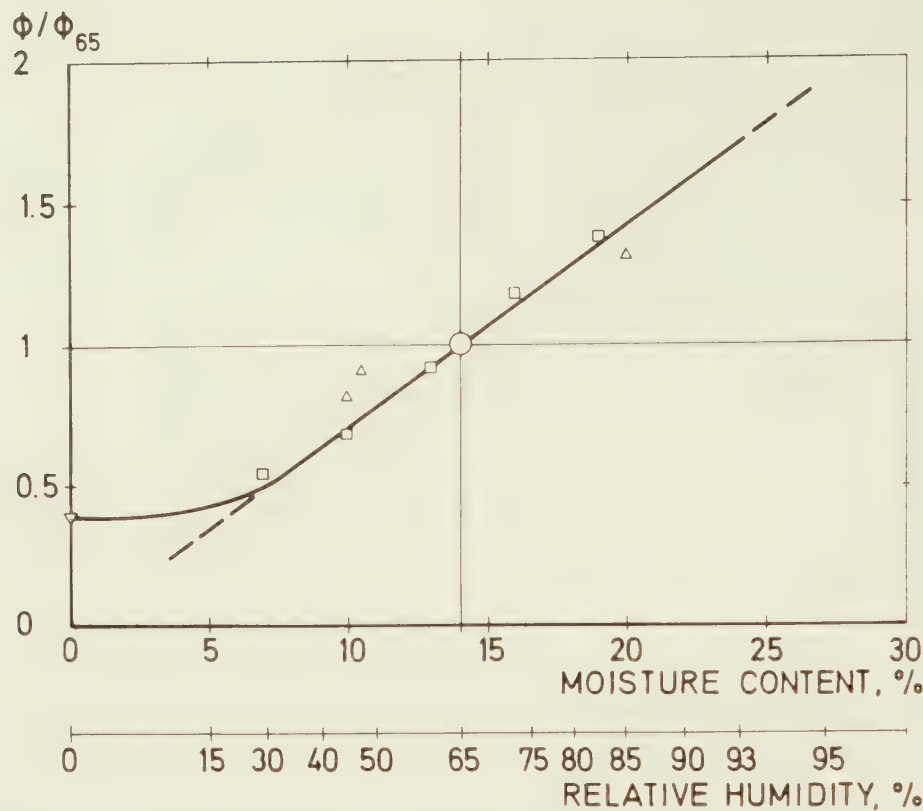


FIG. 7.6 Fractional creep of wood vs. moisture content. The fractional creep at $u = 14\%$ ($\sim 65\%$ RH) is unity. Data from *Becker and Reiter (1970)*, $\square$, and *Perkitny (1966)*, Δ . The straight line is a regression line for 35 experiments by 10 authors, ($6\% < u < 24\%$), calculated by *Wallin (1971)*.

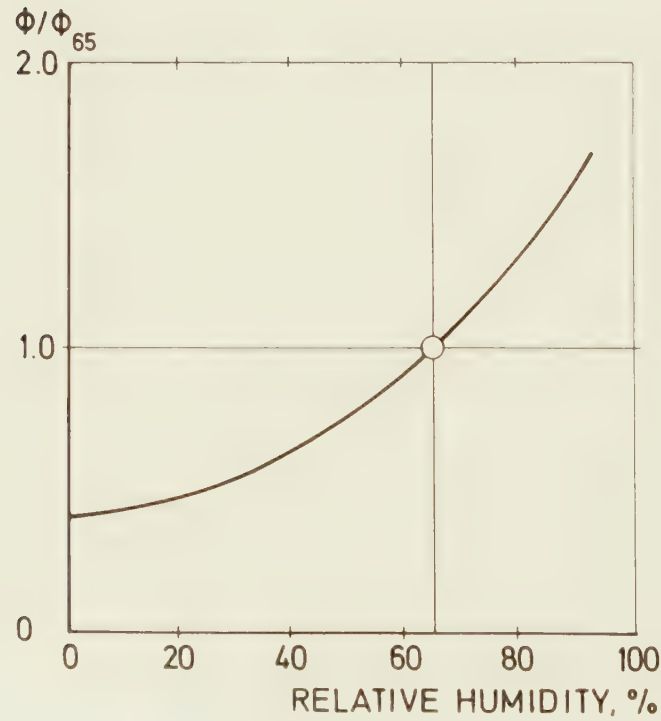


FIG. 7.7 Fractional creep of wood vs. relative humidity. The fractional creep at 65% RH is unity. Same Φ/Φ_{65} values as in FIG. 7.6.

creep values were then plotted vs. moisture content, u . The u values ranged from $u = 6\%$ to $u = 24\%$. No significant interaction between the influences of moisture content, stress or temperature could be seen. Thus, a common regression line for all the values was determined. (The coefficient of correlation was 0.89, which for a set of 35 values gives a very high level of significance for the correlation.) From this regression line the value at $u = 14\%$ ($\sim 65\%$ RH) was defined as unity, and the results at other moisture contents were then plotted relative to this value. The plot is shown in FIG. 7.6.

FIG. 7.6 also shows the data from *Becker and Reiter (1970)* transformed from their original relaxation values (see Section 7.2). These values represent 5 of the experiments included in the calculation of the regression line. They are shown in order to demonstrate the fit of a comprehensive data set to the regression line. FIG. 7.6 also shows data from *Perkitny (1966)*. His values at $u = 10\%$ and 20% are included in the regression analysis. The value at $u = 0\%$ is the only one I have found in the literature at this moisture content. The creep of dry wood is probably not zero as the regression line happens to show, but positive as shown for cement paste in FIG. 7.4. The "trend" curve from $u = 0\%$ to $u = 6\%$ is therefore drawn through Perkitny's value.

It is known that Young's modulus, E_0 , for wood decreases about 12% when the moisture content increases from 5% to 30% (*Thunell (1941)*).

To compare the influence of the RH on the fractional creep of wood to the influence of the RH on the creep of other materials, the relative values Φ/Φ_{65} from FIG. 7.6 are replotted vs. relative humidity in FIG. 7.7. (The mean curve for absorption and desorption in FIG. B.10 is used to convert the data.) The values at 90% RH and higher are very uncertain. The trend in the curve is the same as that for cement paste

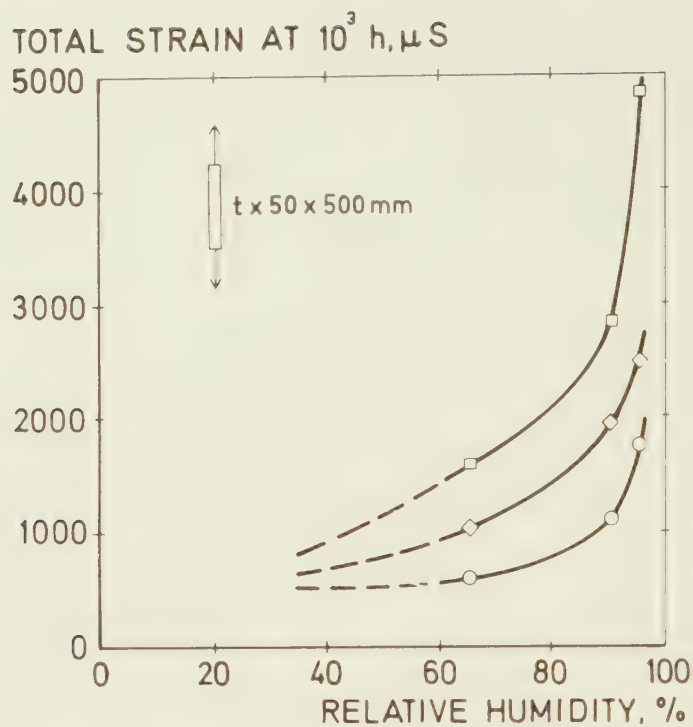


FIG. 7.8 Total strain after 1000 hours vs. relative humidity for wood-based materials. $\sigma/\sigma_B \sim 15\%$. Data from Lundgren (1968).

- 10 mm particle board
- 12 mm panel board
- ◇ 6.3 mm oiltempered hardboard

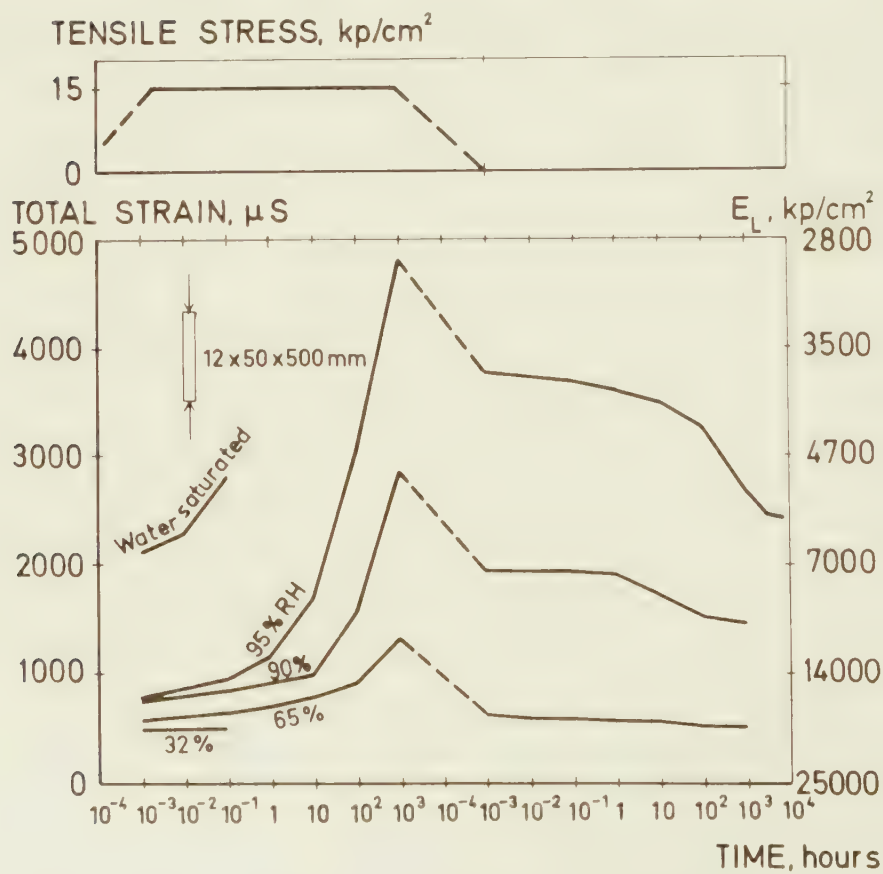


FIG. 7.9 Total strain vs. time for 12 mm panel board in equilibrium with different RH. Load 14 kp/cm² ($\sigma/\sigma_B \sim 15\%$). From Lundgren (1968).

shown in FIG. 7.4.

Lundgren (1968) has examined the long term deformational behaviour of woodbased products (plywood, particleboard and different types of hardboard) at different moisture conditions and stress levels. FIG. 7.8 shows the total strain after 1000 hours under load (tension) for some of the materials. The increases at higher relative humidities are very pronounced.

The moisture content also influences the relative magnitudes of the initial and the creep strain, and the relative magnitudes of the initial and the time dependent recovery. FIG. 7.9 shows total strain vs. time for the panelboards of FIG. 7.8. It is seen that as the RH increases it is the time dependent part of the deformation which increases the most. The initial components are not affected to the same extent. The same kind of relationship was found by *Hansen (1960)* for cement mortar beams. The phenomenon is also illustrated by TABLE 6.1, which shows the fractional recovery increasing with the RH.

For the cementitious materials the presence of water makes it possible for the hydration processes to continue. This is probably true also for steam cured materials. This continued hydration can explain why the modulus of elasticity of cellular concrete increases with time (see FIG. C.4). It may also explain the decrease of the reversible creep components shown for concrete in FIG. 6.8, and for cellular concrete in FIG. 6.9.

To summarize: The effect of increased equilibrium moisture content in a material is to increase basic creep substantially.

7.4 Creep under varying climatic conditions

Structures are usually not situated in a laboratory environment with sophisticated climate controls. In practice the temperature and the relative humidity of

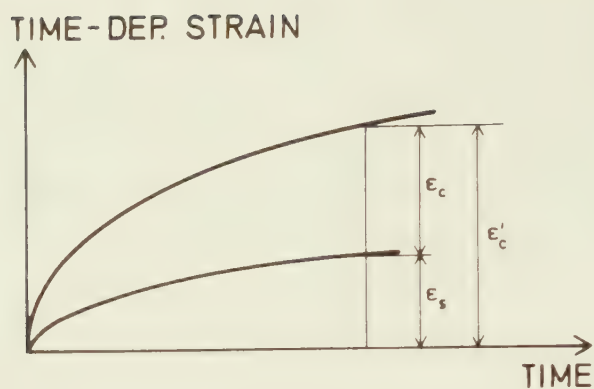


FIG. 7.10 Conventional definition of sorption creep, ϵ_c

ϵ'_c = total time dependent strain

ϵ_s = shrinkage or swelling strain

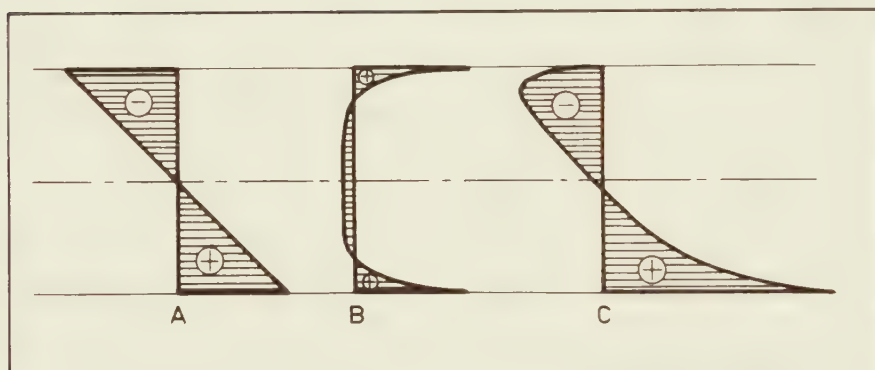


FIG. 7.11 Stress distribution in a beam.

A. Loaded. Moisture equilibrium.

B. Unloaded. Internal stresses caused by drying.

C. Resultant stress distribution (A+B).

the air vary simultaneously. The possible combinations of changing temperatures and humidities are infinite. A general treatment of this problem is therefore very difficult. It may seem as if I have compounded the difficulties by treating creep during the variations of temperature and humidity in the same section, but since they are not independent this is the only realistic approach.

When studying the deformational behaviour of building materials under changing moisture conditions, creep is usually defined as the difference between the total time dependent deformation of a loaded specimen and the shrinkage or swelling of an unloaded dummy placed in the same climate, see FIG. 7.10. This point has been mentioned earlier. Mathematically this procedure is simply expressed:

$$\epsilon_c = \epsilon'_c - \epsilon_s \quad (7.1)$$

This procedure is followed in spite of the fact that shrinkage and swelling are influenced by creep and vice versa. When sorption creep is discussed in the following it is defined in accordance with eq. (7.1). For bending experiments this correction is meaningless, since unloaded dummies do not deform in a flexural mode during sorption. A similar correction to eq. (7.1) can naturally be made for creep during changing temperature.

In practice it is the total deformation which is of interest. In cases where shrinkage occurs together with tension, and swelling occurs together with compression, the total deformation ϵ_e may be of opposite sign to ϵ'_c , depending on the magnitude of the applied force.

Many phenomena arise in a loaded body during change of temperature or humidity.

- * The change of temperature and humidity will cause deformations by itself, namely thermal expansion or contraction and swelling or shrinkage.

- ⌘ These deformations will cause internal stresses which add to the external stress and increase the creep. This phenomenon is described in more detail below.
- ⌘ The internal stresses will cause strain hardening of the material in the same way as external stresses.
- ⌘ The creep rate approaches the basic creep rate for the new temperature and moisture condition.
- ⌘ As mentioned above, a loading will cause changes in temperature as well as in moisture content of a material.
- ⌘ It is known from the science of heat and moisture transfer that a change of temperature will cause diffusion of heat as well as of moisture, and conversely a change of the surrounding relative humidity will cause a diffusion of moisture as well as of heat.
- ⌘ It is known that an increase of temperature or a change in moisture content may cause permanent changes in the microstructure of materials and thereby changes in the deformational behaviour.

When all the above mentioned internal phenomena and the interactions between them are considered, it is not surprising that many experimental results seem to be contradictory and that no common theory for the behaviour of the materials during climatic changes is available.

The creep during climatic changes is usually greater than the creep in constant climate. Many of the phenomena mentioned above probably contribute to this observed behaviour. However, for different reasons it seems as if the presence of internal stresses is a major contributor.

As an illustration of the development of internal stresses and their influence on creep, the stress

distribution in a loaded drying beam shall be treated.

Sketch A in FIG. 7.11 shows the usual triangular stress distribution for moisture equilibrium. Sketch B shows an assumed internal stress distribution in an unloaded specimen during drying. Such a stress distribution may be roughly estimated from curves showing the distribution of moisture in the specimen at a certain time during drying combined with a knowledge of shrinkage as a function of moisture content and with a knowledge of Young's modulus E_0 . The stress distribution will steadily change with time because of the drying and because of the stress relaxation. But the stress profile during a certain time interval may, in principle, be represented by sketch B.

Sketch C shows the combined stresses. The increased stress on the tension side will increase the creep of the outermost fibres. The decreased stress in the compression side will decrease the creep of the outermost fibres there. But since the relation between creep and stress is nonlinear, as shown in FIGS. 6.1-6.4, the increase of the stress on the tension side will increase creep more here than it is decreased on the compression side. Thus, the resultant deflection will increase during the drying compared to the deflection of a beam in basic creep.

The magnitude of the internal stresses depends mainly on the shrinkage and E_0 . The internal stresses can be of the same magnitude as the external stresses. For example, the shrinkage of cellular concrete can be calculated according to $\epsilon_s = |\Delta RH\%| \cdot 10^{-5}$ (Nielsen (1971)). If the drying of the beam is made in 80% RH then $\epsilon_s = 0.20 \cdot 10^{-3}$. With $E_0 = 15 \cdot 10^3$ kp/cm² this yields a stress of $\sigma = 3$ kp/cm². The beams of FIG. 7.12 are subjected to a bending stress of 4 kp/cm², which corresponds to about 40% of σ_B . The beam drying in 80% RH will thus be exposed to an additional stress of about 3 kp/cm², resulting in a total

of 70% of σ_B . From FIG. 6.3 it can be estimated that the creep is at least 3 times larger at this level than at the 40% level. This amount may be sufficient to explain the increase of the creep of this beam during drying. It is seen from FIG. 7.12, that when the beam has reached moisture equilibrium the creep rate is almost the same as for the other beams.

When the signs in FIG. 7.11 are changed it is seen that a similar process occurs during wetting. This may explain why absorption as well as desorption creep in bending is bigger than basic creep.

The internal stress hypothesis was recently shown to be applicable to creep in compression during drying by *Wittmann (1966)*. He used small cylinders of cement paste. In this case the compressive stresses in the central region of the specimen (FIG. 7.11, sketch B) are added to the external stresses and increases the creep. Wittmann used cement pastes with w/c-ratios of 0.30 and 0.65. The cylinders were dried in 40% RH. The maximum values of the compressive shrinkage stresses were calculated to be 20% and 100% of the ultimate crushing strength at 28 days for the two types of pastes.

The stress distribution in FIG. 7.11, sketch C, changes with time, partly because the moisture distribution is changing and partly because creep will result in a relaxation of the highest stresses. A theoretical calculation of sorption creep is therefore rather complicated. One must know the basic creep and its dependence on time, moisture content, temperature and stress. Furthermore, the magnitude of free shrinkage, the modulus of elasticity and its dependence on moisture content and temperature, and the diffusion coefficients for moisture and heat transport must be known. One could probably devise an approximate method by considering space- and time intervals within which the conditions are assumed to be constant.

The increase of creep strain during sorption is due to the increase of the permanent strain component as shown in Section 6.2. This may also be explained in terms of internal stresses. The internal stresses cause reversible as well as irreversible deformation. The reversible strain component disappears when the internal stresses disappear. The irreversible component does not disappear and can be rather large because of its nonlinearity with stress.

Some authors have proposed that the existence of moisture gradients by themselves cause the increase of creep during sorption. *Ruetz (1966)* showed that this could not be the case. He creep tested small hollow cylinders of cement paste during steady state diffusion. He maintained one RH inside and another outside, i.e. the moisture gradient was constant. The creep was the same as in specimens with the corresponding amount of water content equally distributed.

The above mentioned phenomena demonstrate that the main factors governing the development of creep during varying climatic conditions are the changing temperature and moisture gradients in the specimen. These gradients in their turn depend on the heating and the sorption rates, upon the shape and dimensions of the specimen (the volume/surface ratio can be used as a parameter), and upon the microstructure of the material. Perhaps some general rules for the development of sorption and "thermal" creep could be given in terms of pore structure characteristics.

Experiments dealing with creep under varying climatic conditions can be divided into three groups:

- * Uni-directional transport of heat or moisture.
- * Cyclical variation of temperature or RH between two fixed, arbitrarily chosen values.
- * Natural climate conditions, indoor or outdoor.

The three groups of experiments shall now be discussed. Within each group, moisture variations will be

treated first, then thermal variations. Bending, compression and tension will be considered. All modes of loading are unfortunately not represented in all three groups. The most common mode is bending.

It might seem as if it should be possible to give some general rules for the creep behaviour during changing climate, involving for example specimen dimensions or rate of sorption. This is not possible using the present collection of experimental data. However, some rules of thumb will be mentioned in connection with the discussion of each experimental group.

7.5 Creep under unidirectional transport

Unidirectional transport of heat or moisture means that the climatic conditions of the specimens are changing from one state of equilibrium to another. The general observations which can be made from such experiments seem to be the following:

1. The magnitude of creep of a specimen which is exposed to sorption or thermal change during loading is commonly larger than the basic creep of a specimen in the original equilibrium climate.
2. A change in RH influences the rate of creep as long as the moisture content change in the specimen is measurable. Hereafter the creep rate approaches the basic creep rate of a specimen at the final relative humidity. (This means that for a material where the basic creep curve can be described by a power function, the creep curve in a log-log diagram will show an S-shaped transition from the basic creep curve in the original climate to the basic creep curve in the final climate.) A similar rule may be made for thermal changes.
3. The creep rate is greater for small specimens during sorption than for larger specimens. In the larger specimens, however, the effect lasts longer.

Therefore it is not possible to say anything in general about the influence of the specimen size on the magnitude of the creep after the sorption is completed. For concrete the final creep value usually is assumed to decrease with increasing volume/surface ratio of the specimens (see Section 7.6). A similar trend is not found for wood. Here one sometimes even finds increasing creep for increasing volume/surface ratio, see below.

4. A large difference between the initial and the final climate produces a much greater creep rate in the beginning of the sorption than a small difference. But when the climatic difference is small the specimens spend more time in, for instance, the wet initial climate. In that case the final amount of creep may be greater for a small climatic difference than for a large one. Thus, no general rules can be deduced concerning the influence of the difference between the initial and the final climate from the available experimental data.
5. It appears that creep during desorption is greater than during absorption for an equal step in relative humidity.

It is possible, in principle, to deduce rules similar to the ones just outlined for creep behaviour during thermal changes. However, too little experimental data is available at the present time to attempt this.

Some experimental results will now be given to illustrate points 1 through 5.

FIG. 7.12 shows some results from my pilot test on creep of cellular concrete in bending during drying from watersaturation, see Appendix C, and *Nielsen (1968b)*. The drying was done at 43% RH and 80% RH.

The stress levels were 2, 3, 4, 5, and 6 kp/cm². The specimens tested at 5 and 6 kp/cm² all broke within 45 minutes. FIG. 7.12 shows the curves for the 4

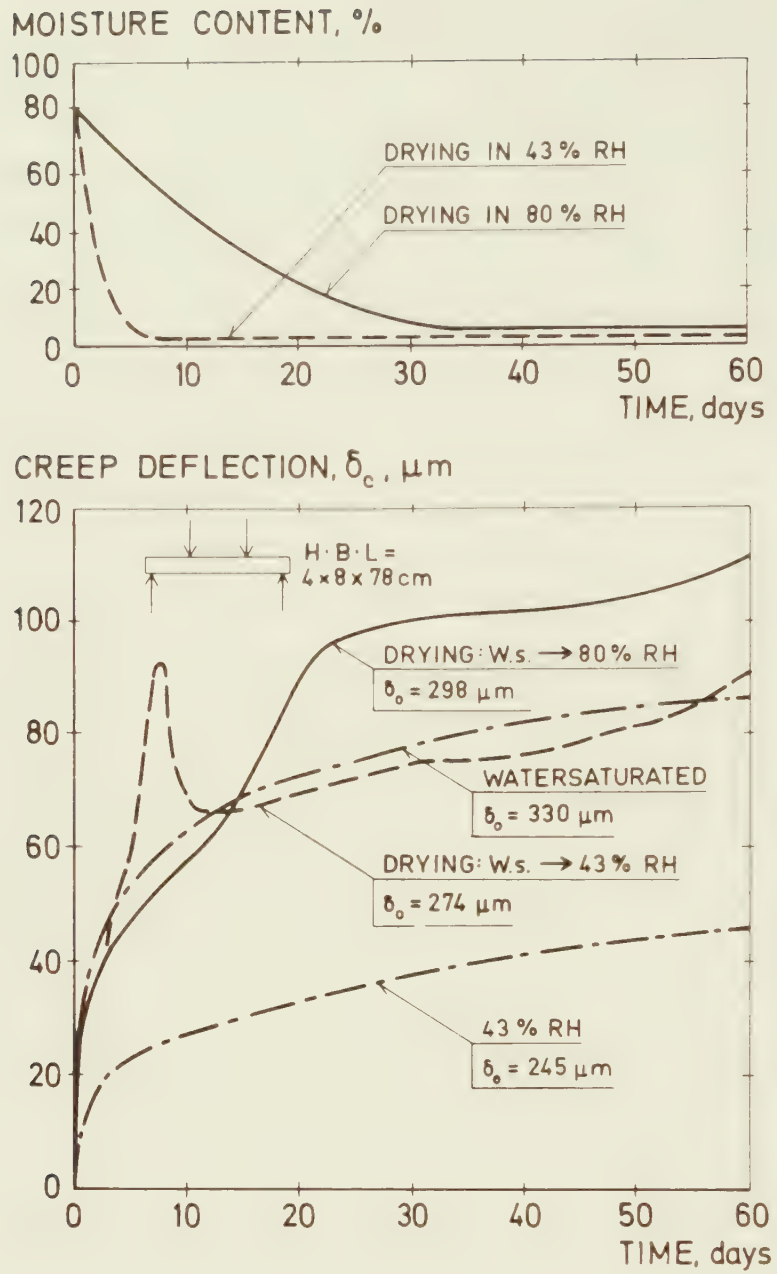


FIG. 7.12 Creep of cellular concrete during drying from watersaturation to equilibrium with 43% RH and 80% RH. Top: Moisture content of unloaded dummies. Bottom: Creep deflection vs. time. For comparison, basic creep curves for watersaturated beams and for beams at 43% RH are shown. Factory D, $\sigma = 4 \text{ kp/cm}^2$. (Nielsen 1968b)).

kp/cm² level. The creep of the specimens dried at 43% RH was fastest in the beginning, but their drying creep curves showed peculiar "humps" during the first 5 to 12 days. These "humps" are caused by gravity flow of water to the underside of the specimen, resulting in different rates of shrinkage at the top and bottom. The unloaded dummies showed similar "humps". The amount of creep for the 43% RH beams is smaller than for the 80% RH beams because the latter are at a high moisture content longer (points 1 and 4).

FIG. 7.12 shows that when the drying is finished the creep rates of the drying beams are about the same as the rates of the two corresponding basic creep curves. When the creep curves are plotted in log-log diagrams the above mentioned S-shaped transition is clearly seen (see *Nielsen (1968b)*), (point 2).

The sorption creep experiments on wood described in Appendix B also gave S-shaped creep curves in a log-log plot. The mean curves and the extrapolations based on the long term experiments are shown in FIG. 7.13. The results are discussed in detail in Appendix B. Some of them shall be repeated here. The small specimens are influenced by the sorption mostly at relatively short times. The final amount of fractional creep is about equal for the small and the large specimens in absorption, but greater for the large specimens in desorption. From FIGS. B.3, B.4, B.6, B.7, and B.8 it is seen that when the moisture change is completed the creep rate is about the same as for a specimen under basic creep at the final climate (points 1, 2, 3, and 5).

The lack of influence of specimen size on sorption creep behaviour of wooden beams was also observed by *Armstrong and Grossmann (1968)*.

Sauer and Haygreen (1968) examined creep in bending of two types of hardboard, both 6.3 mm thick. Some of their results are shown in FIG. 7.14. The dif-

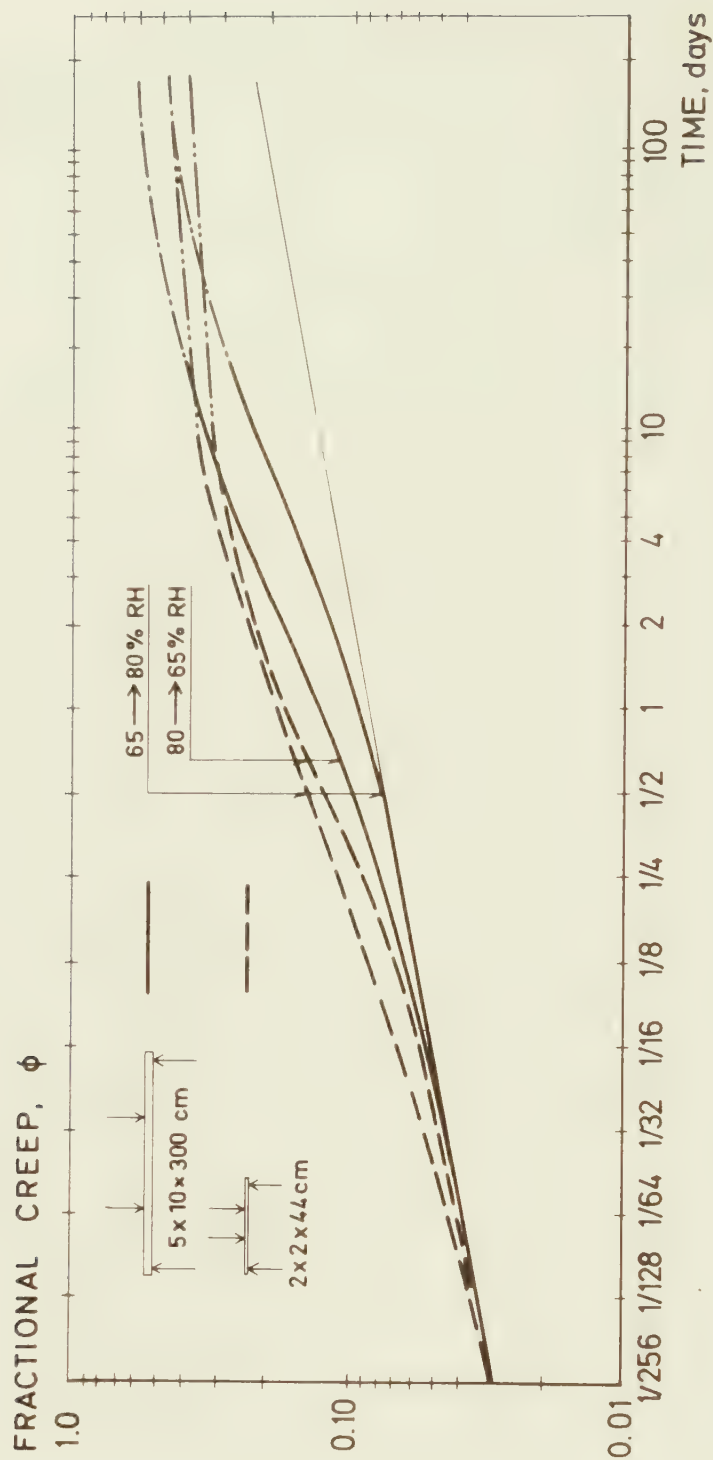


FIG. 7.13 Fractional creep vs. time for pine beams during sorption. Mean curves from FIGS. B.4 and B.5 are shown together.

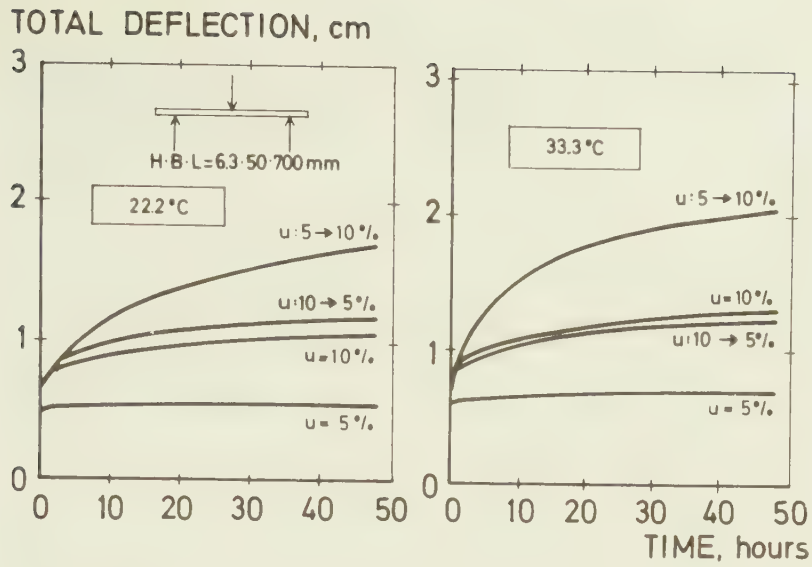


FIG. 7.14 Total deflection vs. time for hardboard loaded during absorption, desorption, and moisture equilibrium. 22.2°C (left) and 33.3°C (right). $\sigma/\sigma_B = 30\%$.

From *Sauer and Haygreen (1968)*.

$u = 5\%$ corresponds to 20% RH

$u = 10\%$ corresponds to $\begin{cases} 50\% \text{ RH desorption} \\ 65\% \text{ RH absorption} \end{cases}$

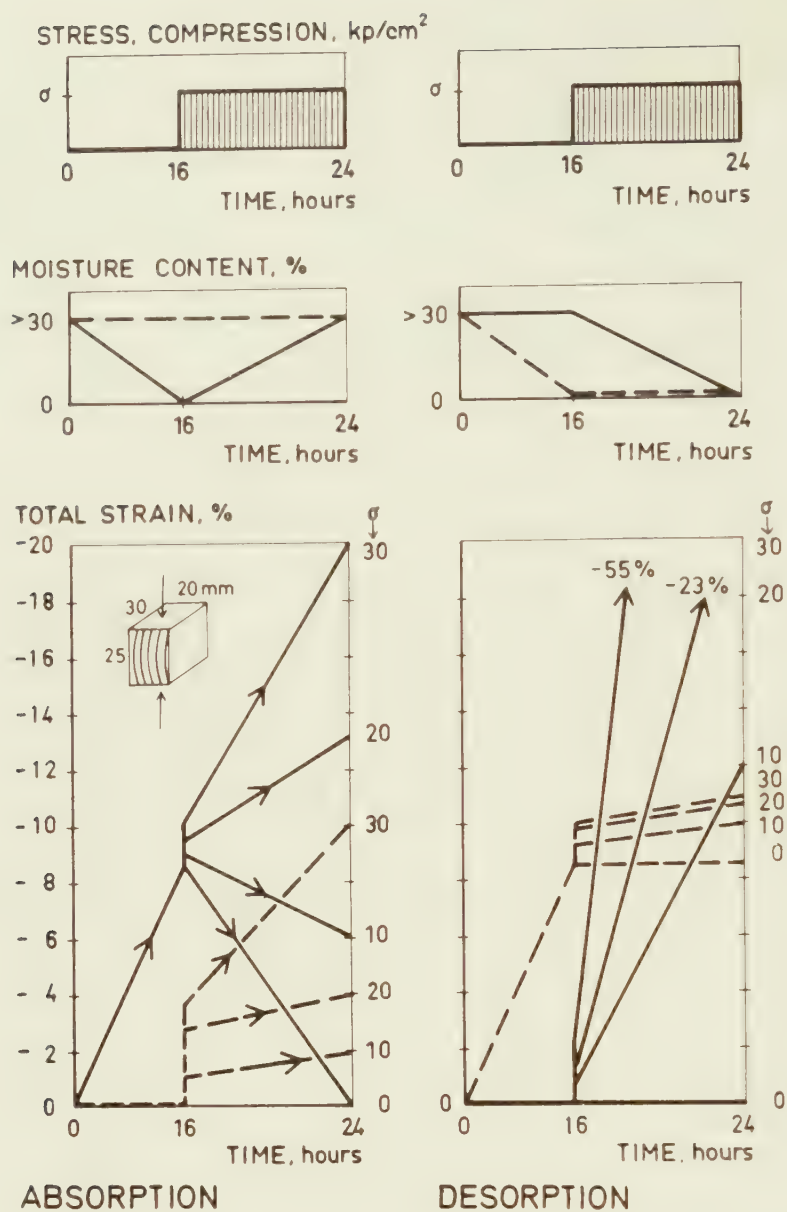
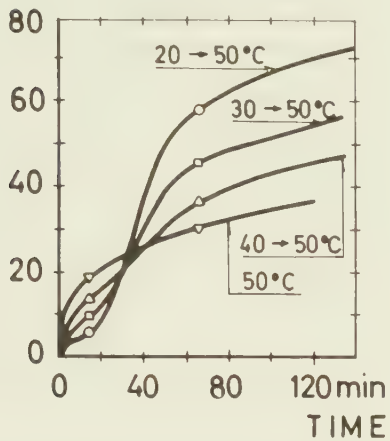


FIG. 7.15 Total strains at discrete times for pine. Straight lines connect data points. Stress and moisture history given in top figures. From *Perkitny (1965)*.

TEMPERATURE, °C



DEFLECTION, 10^{-2} mm



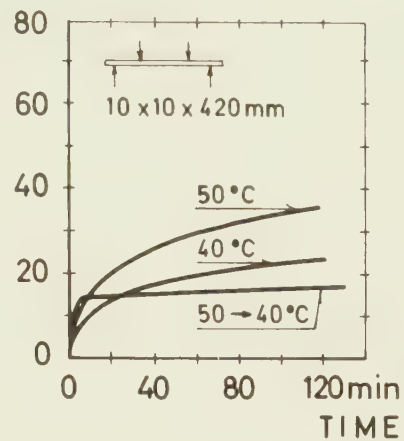
INCREASING TEMP.

A.

TEMPERATURE, °C



DEFLECTION, 10^{-2} mm



DECREASING TEMP.

B.

FIG. 7.16 Creep deflection vs. time for watersaturated wooden beams. Water bath temperatures given in top figures. From *Kitahara and Yukawa (1964)*.

ference between basic and sorption creep is very large. In this case the absorption creep is greater than the desorption creep. This is contrary to what I have observed for wood (points 1 and 5).

Perkitny (1965) has examined sorption creep of wood in compression. Some of his results are shown in FIG. 7.15. He started with wet specimens, which were then dried or kept wet until loading, after which they were dried, wetted or kept at constant moisture content during loading. During absorption, swelling and creep act in opposite directions, but even so the creep was greater during absorption than the creep in constant wet or constant dry condition. During desorption shrinkage and creep act in the same direction and in this case the resultant total strains are extremely large (points 1 and 5).

That a change in temperature during the time under load also increases creep is illustrated by *Kitahara and Yukawa (1964)*. They measured creep of small, watersaturated wooden beams in flexure at constant as well as changing temperatures. (Their results at constant temperatures have contributed to FIG. 7.3.) Some results for changing temperatures are shown in FIG. 7.16. The temperature was changed after the load had been on for 10 min., from 20°C, 30°C, and 40°C to 50°C. The creep of any of the beams undergoing temperature changes was greater than the creep of the beam kept at a constant 50°C. The increase is more pronounced the greater the temperature step. The authors found that the creep curves could be described by the power function, $\delta = A \cdot t^b$, both before and after the temperature change. The factor A increases significantly while b is only slightly diminished during the change. This behaviour is similar to that found for creep during moisture changes as discussed previously. They also examined creep during a temperature decrease from 50°C to 40°C, FIG. 7.16, B. In this case the creep almost stopped, and the creep curve lies under the creep curve at the final tempe-

rature. The significance of this result was not discussed by the authors.

7.6 Desorption creep of concrete

The creep behaviour of concrete is unique in many ways. Its properties change during the loading period due to the continued hydration. It usually undergoes desorption creep, since it is loaded soon after casting and is therefore initially in a moist condition. The drying takes a very long time (years) because of the low diffusion rate of water in concrete.

I will give a qualitative discussion of the creep of concrete (For a detailed review of the creep of concrete see *Neville (1970a)*, among many others.)

Concerning the time function the following shall be pointed out. The power function eq. (5.2) seems to be applicable to basic creep of concrete. *Wittmann (1967)* found that the power function was applicable to creep of sealed specimens of cement paste. Recently, *Kajfasz and Szulc (1970)* found that it was applicable to creep of very old concrete during climatic changes. They obtained rather small values for b ($b \approx 0.1$). The mean curve (FIG. 5.5) given by Wagner, however, does not obey this formula. Wagner's curve shows higher creep rates at short times and lower at longer times. This is partly because of continued hydration, and partly because the curve is for desorption creep. Wagner calculated his curve as the mean of the time functions for all the collected creep experiments. Most of these were carried out during desorption. The concept of a final creep value for concrete has already been discussed in Section 5.2.

The big difference between basic creep and sorption creep is experimentally demonstrated by *Ruetz (1966)* for small cylinders of cement paste loaded in compression. His results are shown in FIG. 7.17. The top

$\epsilon_{ct}/\epsilon_{ct,0\%}$

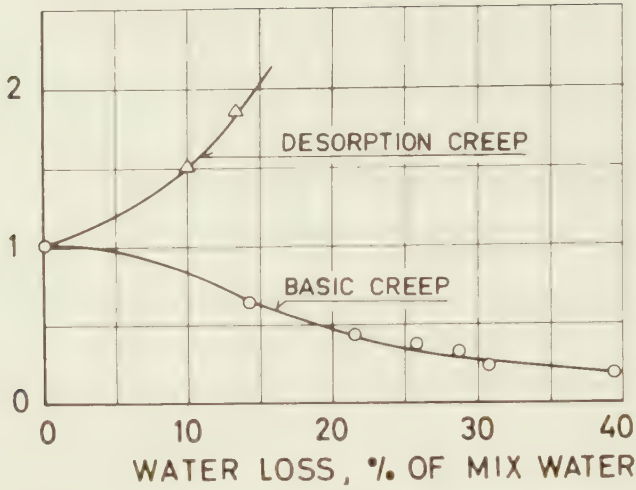
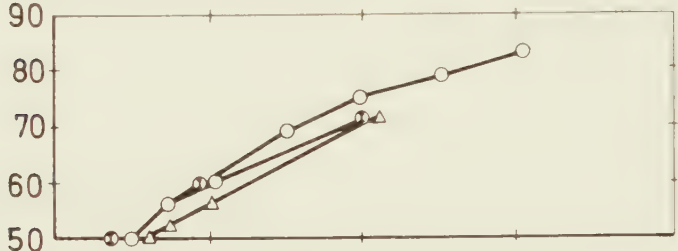


FIG. 7.17 Creep after a certain time vs. water loss for cement paste. The basic creep of a water-saturated specimen is unity. From Ruetz (1966).

RELATIVE HUMIDITY, %



FRACTIONAL CREEP, 1200 days, ϕ

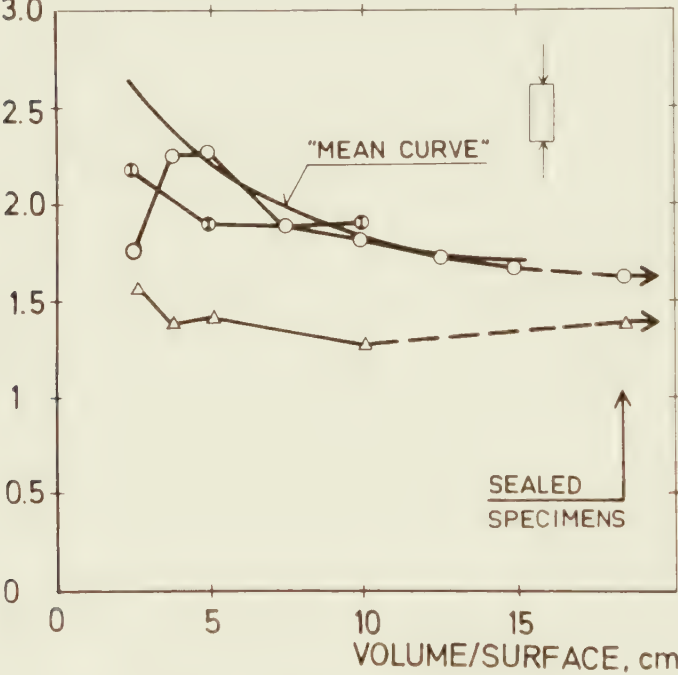


FIG. 7.18 Fractional creep at 1200 days vs. V/S. Concrete drying in 50% RH, 20°C.

Top: RH measured in center of specimens.

- 0 Elgin gravel, cylindrical
 - (I) Elgin gravel, I-shaped
 - Δ Sandstone, cylindrical,
- $\left\{ \begin{array}{l} \sigma = 84 \text{ kp/cm}^2 \\ \bar{E}_0 = 254000 \text{ kp/cm}^2 \end{array} \right.$
- $\left\{ \begin{array}{l} \sigma = 63 \text{ kp/cm}^2 \\ \bar{E}_0 = 120000 \text{ kp/cm}^2 \end{array} \right.$

From Hansen and Mattock (1963).

curve is for sorption creep and the bottom curve is for basic creep under different moisture conditions. The difference between the two curves is very pronounced. The bottom curve is analogous to the curve in FIG. 7.4 (moisture content is related to RH via the sorption isotherm).

Empirical formulas to predict creep have been proposed by *Wagner (1958)*, among others. *Bergström and Nielsen (1969)* reviewed his method. We added a term to Wagner's analysis taking into account the influence of the volume/surface ratio on creep. CEB-FIP, in their recommendations for the design of concrete structures (1970), also recommended that a correction be made for the volume/surface ratio. In both cases the corrections are based on the unique experiments by *Hansen and Mattock (1966)*.

Hansen and Mattock creep tested concrete specimens drying in 50% RH at 20°C. They used two types of concrete aggregate: Elgin gravel and sandstone. Elgin gravel is a normal, rather hard aggregate, consisting mainly of calcite, and the sandstone gravel is a rather soft aggregate, consisting mainly of quartz. They compression tested cylindrical and I-shaped specimens. The volume/surface ratios (V/S) varied as shown in FIG. 7.18.

The fractional creep values at 1200 days for all the specimens are shown in FIG. 7.18 together with the curve named "mean curve" for the Elgin gravel concrete cylinders. The whole family of such "mean" isochronous curves from Hansen and Mattock's paper is the basis for V/S correction mentioned above.

However, FIG. 7.18 shows substantial scatter at low V/S ratios for the Elgin gravel specimens. The mean curve should have been drawn lower at low V/S ratios.

Obviously the result for the Elgin gravel concrete cylinder with $V/S = 2.5$ cm has been regarded as an experimental error when drawing the "mean curve".

This is not necessarily so. This small specimen has dried rather fast. During the drying period the fractional creep is of the same magnitude or larger than that for the larger specimens. After drying to equilibrium with the 50% RH (after about 200 days) it shows a smaller creep rate than the specimens with higher V/S-ratios which are not yet in equilibrium with the surrounding air. Its rate is also smaller than that of the sealed specimen. This last point is demonstrated by the following. The original results by Hansen and Mattock shows that the fractional creep of the specimen with $V/S = 2.5$ cm from 200 days to "infinity" is 27% of the corresponding value for the sealed specimen. This percentage coincides with the findings in FIG. 7.4, which shows that the final fractional basic creep of cement paste at 50% RH is 25% of that at 100% RH.

The effect of the V/S-ratio on the sandstone-concrete is not very pronounced. This observation should possibly be viewed together with the fact that the mean value of the modulus of elasticity for this material is 120000 kp/cm^2 , or only half the value of the Elgin gravel concrete. The magnitude of the modulus of elasticity influences the magnitude of the internal stresses which are developed during the drying; the smaller E_0 , the smaller the internal stresses.

Taking into account all of Hansen and Mattock's results it appears questionable to apply their "mean" curve to correct for the effect of V/S-ratio on creep.

This example demonstrates that the dependence of creep on the specimen size can not be said to be fully understood for concrete, nor for any other material.

7.7 Cyclical variations of climate

By cyclical variations is meant preprogrammed, fairly rapid fluctuations in temperature or relative humidity between two limiting values.

- * The general conclusions which can be drawn from such experiments are that the increase of creep during desorption cycles is larger than the increase during absorption cycles, and that creep defined according to eq. (7.1) in all cases is larger than the basic creep in any of the limiting climates.

Experiments where the relative humidity of the air surrounding the specimens vary are rather common.

Hansen (1960) tested cement mortar beams, *Bernhard (1967)* tested concrete cylinders, *Eriksson and Norén (1965)* tested extremely thin wood samples in tension, *Hearmon and Paton (1964)* tested small beams of beech wood, and *Bethe (1969)* also tested small wooden beams.

Bernhardt (1967) has also investigated the creep of concrete during cyclical variations of the temperature between 20°C and -5°C . The resulting fractional creep values were larger than creep at a constant 20°C .

It must be mentioned that it evidently is possible to deform a hygroscopic (tough) material considerably by changing the climate in a suitable way. The experiment by *Hearmon and Paton (1964)* on wood is a good example. They used a stress equal to 38% of the rupture stress, and cycled the RH between 0% and 93%. The fractional creep reached a value of 25 at 28 days, whereafter failure occurred (FIG. 7.19).

Even unloaded specimens can be brought to failure by subjecting them to extreme changes in climate as shown by *Malinowski et al. (1966)* for cellular concrete. Specimens of size 6 x 6 x 18 cm were kept in water for 4-5 days at 20°C , and then dried for 4-5 days at $80-85^{\circ}\text{C}$. After 6 cycles of this treatment failure occurred. These phenomena can be qualitatively explained as fatigue ruptures caused by internal stresses.

When the hypothesis concerning the influence of inter-

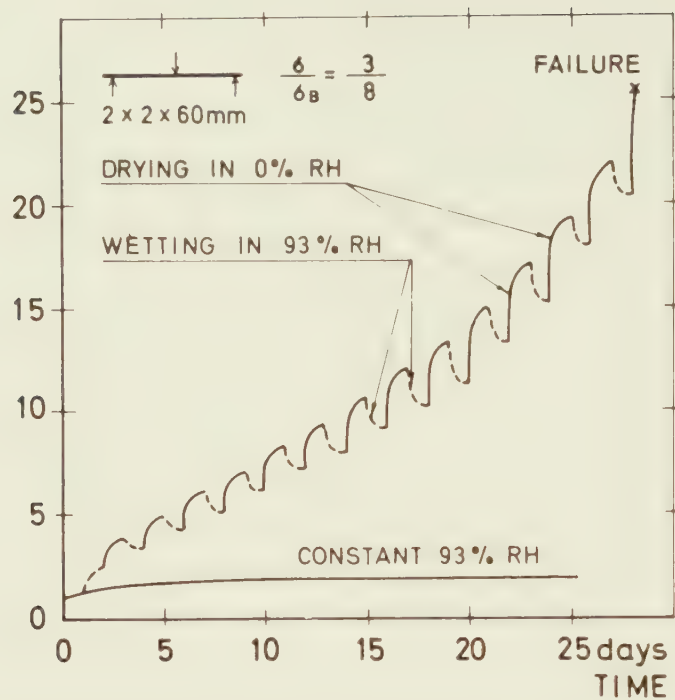
FRACTIONAL CREEP, ϕ 

FIG. 7.19 Fractional creep vs. time for beech beams.
From *Hearmon and Paton (1964)*.

nal stresses on creep is brought to mind it seems reasonable to expect that the specimen size should influence the magnitude of creep during cyclical climatic variations. The creep of small samples ought to be greater than the creep of large samples. No systematic data are available on this point.

7.8 Variations of climate in practice

In this section I will discuss the variations in climate which are found in practice, outdoors and indoors, and their influence on creep.

To illustrate the magnitude of the variations FIG. 7.20 shows the mean values of temperature and relative humidity outdoors and indoors in Stockholm during the year. Variations between days, and between day and night, are in addition to the variations of the daily mean values shown here.

The influence of the climate on creep is, as discussed earlier, a combined effect of levels as well as changes of temperature and humidity. Considering the findings in previous sections one may expect the following behaviour under practical conditions.

1. The creep outdoors should be greater than the creep indoors because of the higher level of RH outdoors and because of the greater changes in temperature and RH.
2. The creep rate outdoors may be expected to increase during the summer due to the higher temperatures, and to decrease during the winter due to the lower temperatures.
3. The creep rate indoors may be expected to increase during the summer due to the higher RH and to decrease during the winter due to the lower RH.

These expectations are confirmed by experiments by, among others, *Kingston and Armstrong (1951)* on solid

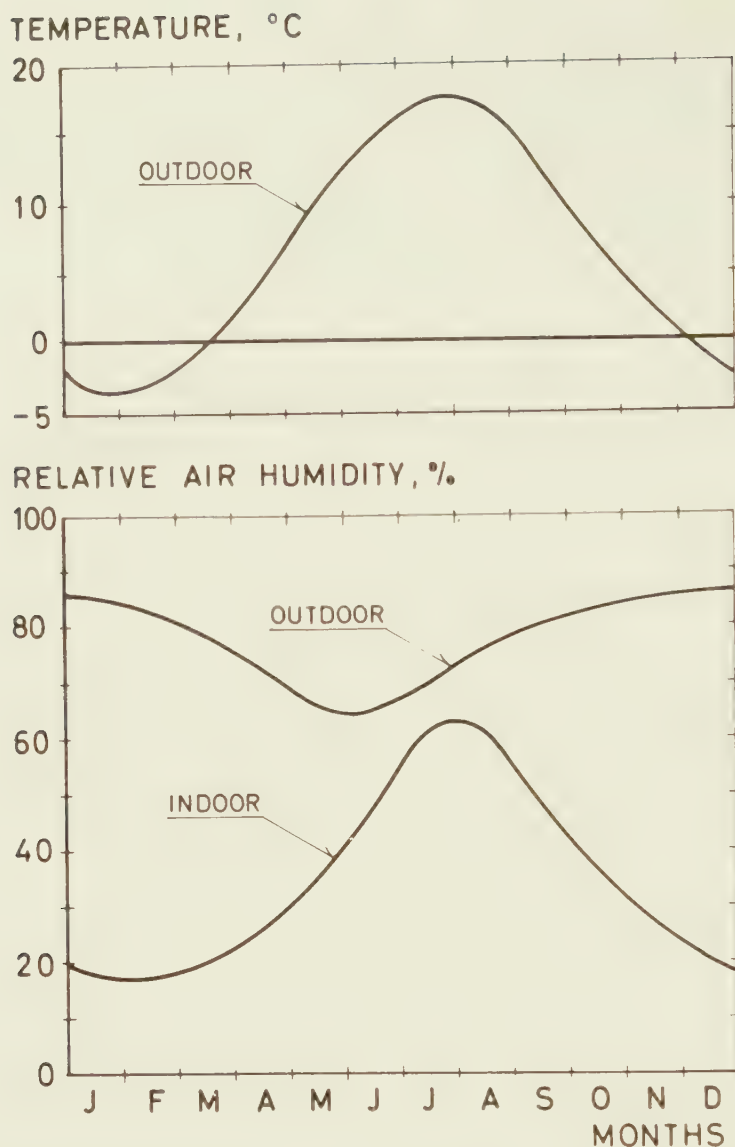


FIG. 7.20 Temperature and relative air humidity variations during the year. Outdoor values are mean values for the Stockholm area. Indoor values of RH are calculated (*Adamsson et al. (1970)*).

SEASON

W	SUMMER	WINTER	SUMMER	WINTER	SUMMER	WINTER	SUMMER
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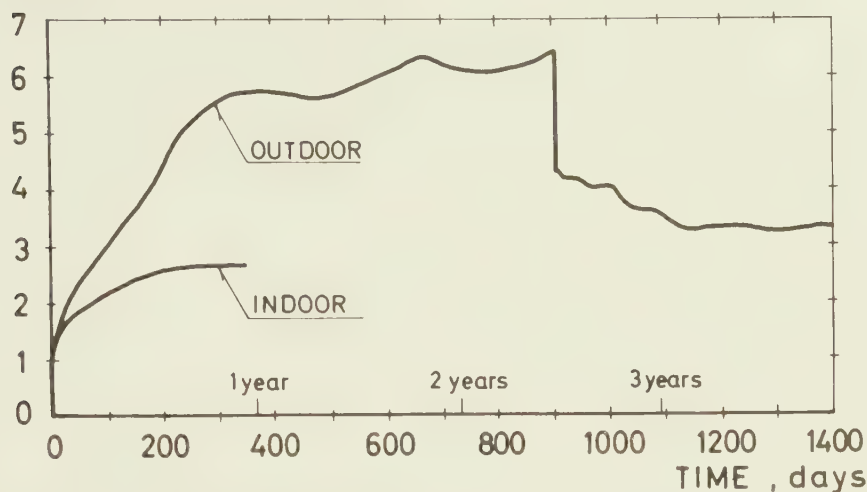
FRACTIONAL DEFLECTION, $\phi+1$ 

FIG. 7.21 Total deflection divided by initial deflection ($\phi+1$) vs. time for ash beams loaded in the green state. The seasonal influence is seen. From *Kingston and Armstrong (1951)*. (Beam No. 117). Bending stress 270 kp/cm^2 . Beam Cross Section: $3 \frac{1}{4} \text{ in. square} \sim 8 \times 8 \text{ cm}$.

SEASON

	W	S	W	S	W	S	W	S	W	S	W	
YEAR	62	1963		1964		1965		1966		1967		68

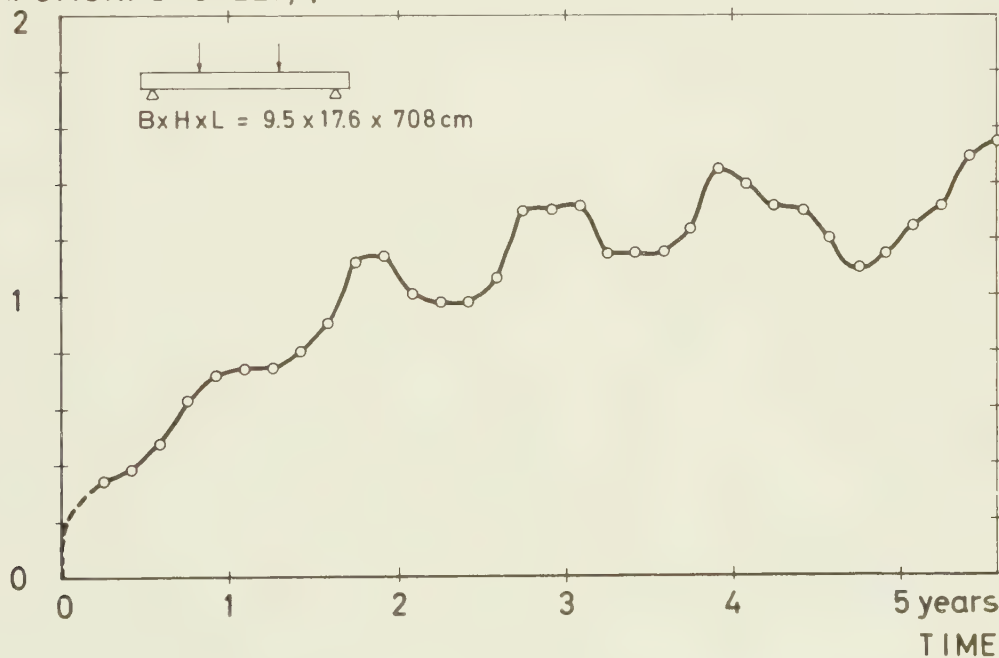
FRACTIONAL CREEP, ϕ 

FIG. 7.22 Fractional creep vs. time for glue-laminated wooden beams tested outdoors. Bending stress 84 kp/cm^2 . Starting moisture content 12%. From *Saarelainen (1969)*. (Beam No. 4).

SEASON

S	WINTER	SUMMER	WINTER	SUMMER	WINTER	S
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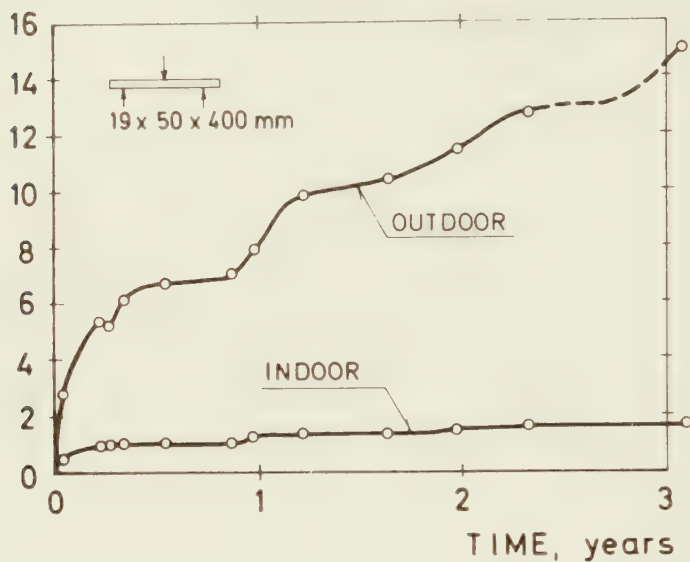
FRACTIONAL CREEP, ϕ 

FIG. 7.23 Fractional creep vs. time for particle board tested outdoors and indoors. $\sigma/\sigma_B = 10\%$. The beams were in equilibrium at 65% RH, 20°C, before testing. From *Kratz (1969)*.

wood, *Saarelainen (1969)* on glue-laminated wooden beams and *Kratz (1969)* on particle board.

The experiments by *Kingston and Armstrong (1951)* were carried out on initially green beams of ash. The cross section of the beams was about 8 cm square. It was a rather big investigation which also showed how varying experimental creep values can be. To show the influence of the climate I have redrawn two of their curves in FIG. 7.21. The fractional creep of the indoor beams is lower than that of the outdoor ones. We see that during the summer period September-March (Southern Hemisphere) the creep of the outdoor beam increases and during the winter period it stops or decreases. The seasonal influence on the recovery curve is not so clear, but the climatic variations influence the recovery even $1\frac{1}{2}$ years after unloading. For the outdoor beams Kingston and Armstrong also found that the creep rate was much greater during the day than during the night. They presumed this to be due to temperature effects.

The fractional creep values found by Kingston and Armstrong are rather high compared to normal values for wood. This must be due to the sorption and to the high stress level (bending stress 270 kp/cm^2). In addition, the outdoor beams were exposed to the high summer temperatures.

The investigation by *Saarelainen (1969)* is important to the evaluation of long term deformations of glue-laminated beams. Twelve Finnish beams of constructional dimensions were tested for more than 5 years at stress levels close to the allowable. The beams were outdoors. Different wood grades were used. A representative result is seen in FIG. 7.22. Increased creep during the summer seasons (March-September) and decreased creep during the winter seasons are seen. Some irregularities arise during 1967-68, but they can not be explained without further information about temperature and humidity. In one of the series

of six beams, three beams were sealed with plastic foil and three beams were unsealed. From the original diagrams a decrease in the deflection of the sealed beams during the first year can be seen, but after that the sealing has no significant influence.

Kratz (1969) investigated the creep behaviour of particle board in flexure. The main purpose of his experiment was to clarify the influence of different polymer binders, impregnations and specific weights on the creep. Two of his curves are shown here in FIG. 7.23 to illustrate the seasonal influence on creep. The stress intensity was only 10% of the bending strength.

The beams were brought to equilibrium at 65% RH before loading. In FIG. 7.23 the points are the experimental results, while I have drawn the curves. The marking of the seasons is made on the basis of dates given by *Kratz*. Here, too, the increased creep rate during the summer and the decreased rate during the winter is evident. Even the indoor samples are influenced by the change of season. This may be due to the increased relative humidity during the summer.

Other examples of the influence of climatic changes are shown by the different creep investigations on cellular concrete. Most of the creep curves for reinforced elements of cellular concrete in FIG. C.1 show much greater creep than I found in my investigation of basic creep of cellular concrete. This may be because the reinforced elements were tested in laboratories without any climate control. Some of them were loaded before they were in climate equilibrium. *Schäffler (1960)* shows in his report the variation of temperature and RH during the test. The increased creep during climatic changes are seen clearly here.

In the experiments just discussed, only the temperature and the relative humidity have varied. For roof constructions, increased load due to snow during the win-

ter season may also increase the creep. The magnitude of this extra load is treated on a statistical basis by *Åkerlund (1970)*.

A very well known problem to creep research workers is that of maintaining a constant climate during testing. Fortunately many accidents happen and from these we can also learn something about the influence of climatic changes on creep. Sudden changes in temperature or relative humidity most often cause increased creep, see FIG. 6.13. *Perkitny (1965)* points out that even changes as small as $\pm 1\%$ RH clearly influence the development of the creep of compression loaded wood specimens. His statement is based upon work with small specimens.

However, the specimen size must influence the sensitivity of creep to moisture and temperature changes. Large specimens are not influenced to the same extent as small ones. But, as mentioned in Section 7.7, no systematic data are available on this point.

The main conclusions which may be drawn from the preceding chapters are divided into two groups: Section 8.1 deals with general descriptions of creep behaviour. Section 8.2 gives recommendations for future research.

8.1 General descriptions of creep behaviour

The following points are considered: methods of presenting creep data, the time functions, and the influences of stress and climate on creep.

Creep data can be presented in many different ways (see TABLE 4.1). The presentation in term of the creep modulus E_L , calculated according to eq. (4.10), may be regarded as the most convenient at our present state of knowledge:

$$E_L(t) = E_0 / (1 + \Phi(t)) \quad (4.10)$$

This representation is particularly useful for design calculations which usually are made in terms of moduli. Data from creep research may also be given in this form, since Φ values are easy to obtain.

Among the many empirical creep time functions the power function is found to be the most useful. It may be used in either of the forms:

$$\epsilon_c(t) = A \cdot t^b \quad (5.2)$$

$$\Phi(t) = \Phi_1 \cdot t^b \quad (5.3)$$

The power function describes virgin basic creep almost exactly. It can also be used to approximate virgin creep during fluctuating climatic conditions, as for example is found in and around buildings. The b values are commonly in the range 0.1 to 0.4, more seldom up to 0.7. The b values depend upon the definition of the initial strain.

Equations (4.10) and (5.3) may be combined to yield the following formula for the creep modulus ($\Phi_1 = \Phi_y$):

$$E_L(t) = \frac{E_o}{1 + \Phi_y \cdot t^b} \quad (8.1)$$

where t is in units of years, and Φ_y is the fractional creep after one year.

This formula can be used in design to determine virgin basic creep and virgin creep during fluctuating climatic conditions. E_o is wellknown and Φ_y and b can be estimated using data in the literature. TABLE 8.1 lists a number of experimentally determined values for these three parameters for different materials in different climates. The values refer to stress levels where linearity between stress and strain can be assumed.

One formulation of the linear viscoelastic theory describes the time dependent deformations of materials in terms of rheological models (consisting of linear springs and dashpots). This approach is considered. The rheological model approach can be generalized by considering an infinite number of Kelvin elements. Their properties are characterized by a spectrum of retardation times. This general formulation is used as a tool in materials science to study the relationship between microstructure and deformational behaviour.

The Boltzmann superposition principle may be viewed as a consequence of linear viscoelastic theory. The principle has been discussed. It is pointed out that in order to apply the Boltzmann superposition principle to strain hardening building materials it is necessary to separate the delayed elastic and the viscous components of strain. The principle may then be applied to each component individually using proper descriptions of the strain hardening behaviour. The variation of virgin basic creep with stress is

best described by a sinh function. Linearity may be assumed up to 30% of the crushing strength.

Stress influences the different stress components differently: The hypothesis has been made that the time dependent recovery always amounts to a certain fraction of the initial recovery. The fraction depends on the type of material and the climatic condition. Furthermore, there appears to be a linear relationship between stress and the recoverable strain components at all stress levels. Thus, the nonlinearity between creep and stress seems to be due to the nonlinear variation of the viscous component with stress.

Temperature and humidity influence creep in two ways: Firstly, basic creep is greater at higher levels of temperature and equilibrium moisture content than at lower. Secondly, the appearance of temperature or moisture gradients in the specimen during creep will produce increases in the creep rate. In the case of unidirectional transport this increase disappears when the specimen has reached climatic equilibrium. Hereafter the creep rate seems to become equal to the basic creep rate in the new climate. The increase in the creep rate appears to be caused mainly by an increase in the rate of the irreversible strain component.

The present state of knowledge does not allow one to formulate general rules concerning the influence of specimen size or sorption rate on the total amount of creep.

The observed scatter in the total deformations of specimens selected at random from normal production runs has the same magnitude as that found for strength parameters. Coefficients of variation of 10 to 15% are not unusual. If E_L is not determined directly, but is computed using combinatorial formulas, one must expect greater scatter.

Future research projects may be divided into two, frequently overlapping, categories. The first one concerns an empirical description of creep behaviour under different climatic conditions. The second one aims at a fundamental understanding of the creep processes. The following research programs are recommended:

- * Standardization of experimental procedures so that one can define the "initial" strain component in a consistent way.
- * Systematic collection of virgin creep data under different climatic conditions, for use with eq. (8.1). Such a program will "map out" the virgin creep behaviour of building materials.
- * The program mentioned above may lead to an empirical method to predict creep using eq. (3.2). Such a method would be analogous to the one presently applied to concrete (*Wagner (1958)* and *Bergström and Nielsen (1969)*), and to wood (*Wallin (1971)*).
- * Studies of the extent to which superposition using the Dischinger equation, eq. (6.9), may be applied to other building materials than concrete. It also appears that this method should be applicable to sorption creep and to nonlinear creep if proper expressions for the permanent strain are found.
- * Fundamental studies on the causes of creep, using physical and chemical methods.
- * Studies with particular emphasis on sorption creep and creep during changing temperatures, combined with theoretical analysis of the "internal stress hypothesis".

TABLE 8.1. EXAMPLES OF PARAMETER VALUES IN THE EQUATION $E_L = E_O / (1 + \phi_y t^p)$

Material	Temp. °C	u or RH %	Expo- sure	$E_O \cdot 10^{-5}$ kp/cm ²	ϕ_y	b	Quoted or calculated from
Concrete	~ 20	-	S, C	2.0-3.5	0.5-2.5	o)	Wagner (1958)
Concrete	6-20	50 < RH < 90	S, C	-	0.8-2.5	0.10-0.11	Kajfasz (1970)
Cellular concrete	~ 20	RH = 43	E, B	0.19	0.2-0.3	0.22	Nielsen (1968b)
Cellular concrete	~ 20	u = 85	E, B	0.16	0.2-0.4	0.20	Nielsen (1968b)
Cellular concrete	~ 20	55 < RH < 85	S, B	-	0.5-1.3	0.33	Schäffler (1960)
Calcium Silicate Brick Masonry I	20	RH = 65	S, C ^{*)}	0.41-0.53	1.6-1.8	0.35-0.4	Meyer et al. (1969)
Calcium Silicate Brick Masonry II	20	RH = 65	S, C ^{*)}	1.11	1.2	0.7	Meyer et al. (1969)
Gluelaminated wood	os	os	S, B	-	~ 0.7	~ 0.33	Saarelainen (1969)
Gluelaminated wood	~ 20	10 < u < 12	E, B	-	0.3-0.4	0.28-0.33	Littleford (1966)
Timber and gluelaminated wood	os, i	os, i	S, B	-	~ 1.3	0.3	Nielsen et al. (1970)
Softwood #	20	RH = 65	E, B	-	0.2-0.6	0.15-0.25	Wallin (1971)
Pine 1, 1 kp/cm ² (!)	20	RH = 95	E, T	0.022	~ 7.0	~ 0.2	Lundgren (1968)
Plywood, 5 mm	20	RH = 65	E, T	0.12	~ 0.4	~ 0.40	Lundgren (1968)
Particle board, 10 mm	20	RH = 65	E, T	0.3	~ 0.4	~ 0.3	Lundgren (1968)
Hardboard, 1/8"	20	RH = 65	E, T	0.18	~ 0.5	~ 0.40	Lundgren (1968)
Particle board, 19 mm	i	i	S, B	-	~ 1.2	~ 0.25	Kratz (1969)
Particle board, 19 mm	os	os	S, B	-	~ 12.0	~ 0.40	Kratz (1969)
Aluminium, AlZnMgCu	20	-	E, T	-	~ 0.2	~ 0.27	Weller et al. (1964)

Abbreviations: S = Sorption, E = Equilibrium, C = Compression, T = Tension, B = Bending
os = outside, sheltered, i = inside, ^{*)}No correction for shrinkage.
o) Special time function, see FIG. 5.5.

AN EXPERIMENTAL STUDY OF BASIC CREEP OF WOOD

This appendix deals with some experimental results obtained by *Cederberg and Danielsson (1970)* in connection with their master thesis. The results have also been treated by *Åkerlund (1970)*, who together with me supervised the work.

The importance of accurately determining the initial deformations was pointed out in Sections 5.1 and 5.3. A proper experimental procedure must be designed and followed consistently. Cederberg and Danielsson read the initial deformation when the dial on the gauge stood "still". This procedure is somewhat arbitrary and produced variations. In addition, a systematic error may have been introduced because their two measuring devices were different, see FIG. A.1. For these reasons I have reconsidered the results and reached some conclusions different from those in the previous reports.

A.1 Summary of the experiments

The purpose of the experiments was to investigate the influence of the following variables (see TABLE A.1) on the basic creep of wood:

- * Relative air humidity: 65 and 80%.
- * Wood species: Pine and spruce.
- * Construction grade: T 200 and T 300, according to the Swedish Standard.
- * Stress level: T 200: 80 and 160 kp/cm²
T 300: 100 and 200 kp/cm²

The stress values were 1 and 2 times the allowable design stresses.

- * Specimen size: 2 x 2 x 44 cm and 5 x 10 x 300 cm.

The specimens were sampled at random from a lumberyard. The results, therefore, can be taken as representative of those found in practice. The small spe-

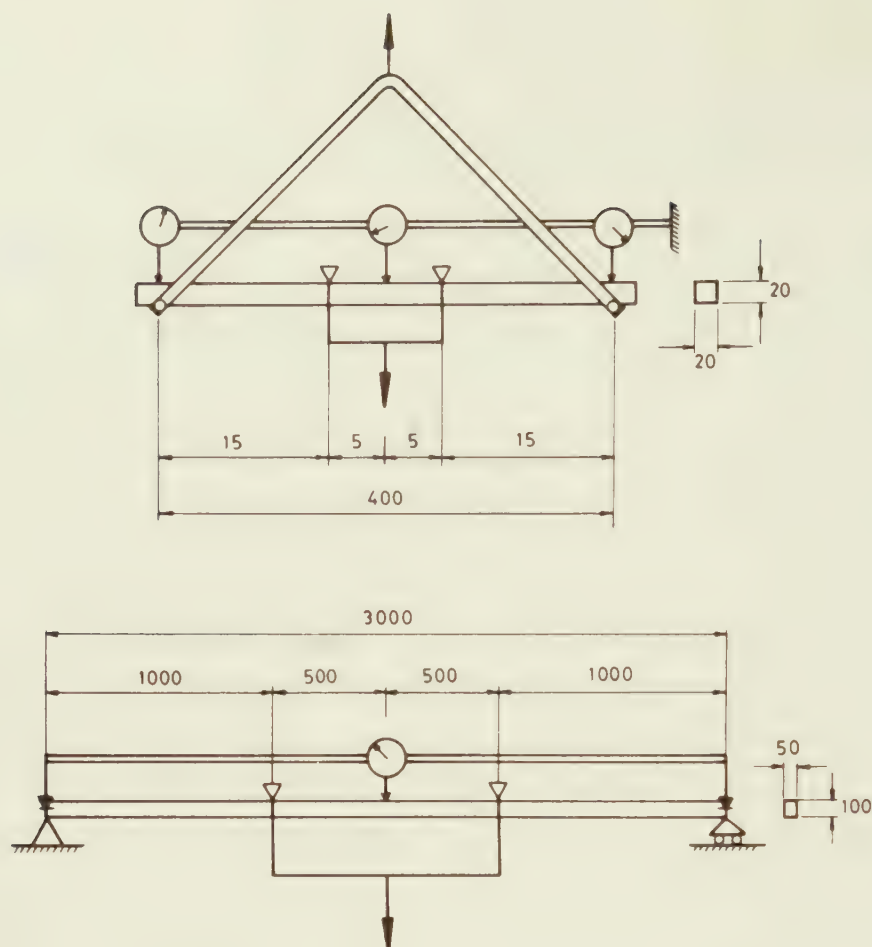


FIG. A.1 Devices used to measure creep of wood in bending. Two different specimen sizes. (*Cederberg and Danielsson (1970)*). All dimensions in mm.

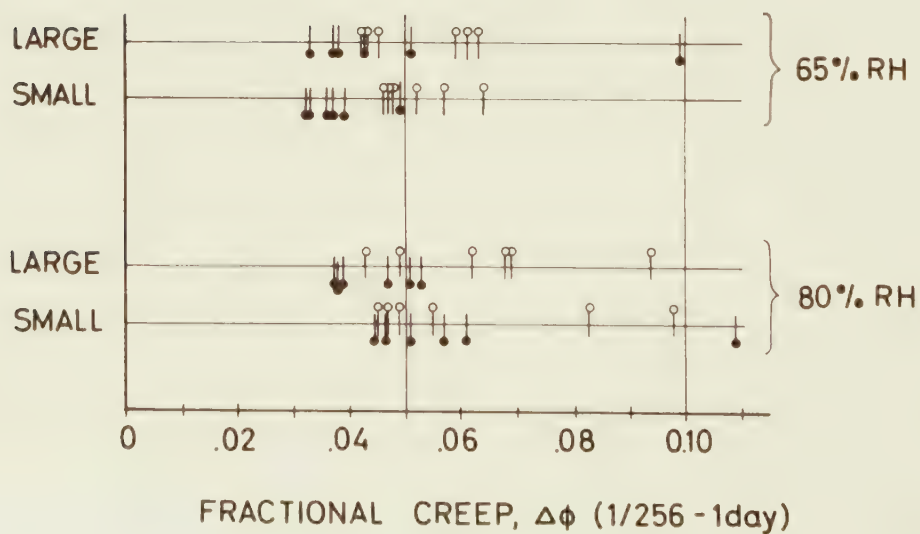


FIG. A.2 Fractional creep of two wood species and two specimen sizes at two relative humidities. (○ = Pine, ● = Spruce).

cimens were without major faults, while the large specimens contained knots and faults as allowed in the Swedish Standard for T 200 and T 300.

The experimental set-ups are shown in FIG. A.1. The deformations of the beams were measured when the dial stood "still" soon after loading, and then at 1/256, 1/128, 1/64, 1/32, 1/16, 1/8, 1/4, 1/2 and 1 day.

A.2 Results

I have used the fractional creep values calculated by Cederberg and Danielsson. The creep behaviour of the two different size beams can then be compared directly. However, for the reasons mentioned above the increase of fractional creep between 1/256 day and 1 day ($\Delta\Phi(1/256 - 1 \text{ day})$) is used to represent the creep behaviour in the present treatment. In this way one avoids most of the error introduced by the uncertain reading of the initial deformation.

$\Delta\Phi(1/256 - 1 \text{ day})$ for all the specimens is shown in TABLE A.1. The same data is plotted in FIG. A.2. The diagram shows the influence of relative humidity, species and specimen size.

The influences of relative humidity, construction grade and stress is seen in FIG. A.3. The influences of relative humidity and stress for the small beams are seen more clearly in FIG. 6.7.

The dependence of fractional creep on stress intensity, σ/σ_B for both specimen sizes is shown in FIG. A.4.

A.3 Conclusions and discussion

The following conclusions can be drawn considering FIG. A.2.

- * Small and large specimens have the same fractional creep.
- * If the extreme values (0.099 and 0.109) are disregarded, it appears that the spruce specimens

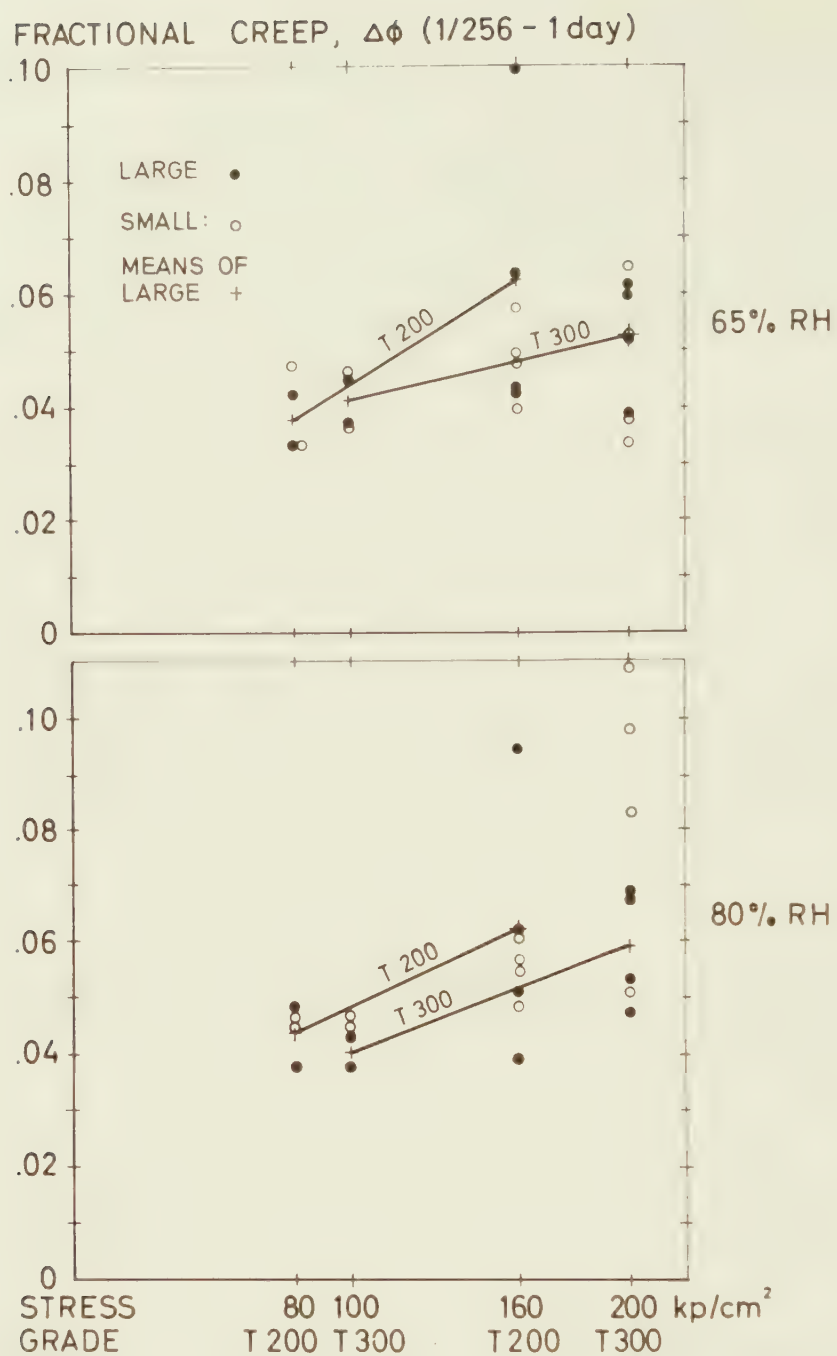


FIG. A.3 Fractional creep at the two levels of relative humidity, as function of stress and wood grade. The lines connect the mean values of the large samples.

have a smaller fractional creep than the pine specimens.

This last point agrees with the variation of the ratio E_o/r_{ou} , as shown by *Åkerlund (1970)* (E_o is the modulus of elasticity and r_{ou} is the density (dry weight divided by wet volume)). However, when all the specimens are considered the significance of the effect of species is not very great.

The following conclusions can be drawn considering FIG. A.3.

- * The fractional creep increases with increasing stress.
- * The fractional creep of both small and large samples is about the same at 80 and 100 kp/cm². At 160 kp/cm² the fractional creep of some of the large beams appears greater than that of the small beams, certainly because of the knots in the large members, T 200. At 200 kp/cm² there seems to be no difference between large and small samples.
- * There is no difference between the fractional creep of T 200 and T 300 specimens at the low stresses. The T 200 specimens creep a little more than T 300 specimens at the high stresses. The statistical significance of this is questionable.

To eliminate the influence of the knots the fractional creep of the small specimens have been replotted as function of stress in FIG. 6.7. From this graph the following conclusions may be drawn:

- * The fractional creep does not increase from 80 to 100 kp/cm², but increases significantly at 160 and 200 kp/cm². These latter stress levels correspond to approximately 27 and 33% of fracture stress. This effect is less noticeable at 65% RH than at 80% RH.
- * The amount of fractional creep of the small specimens is greater in 80% RH than in 65% RH at any stress level.

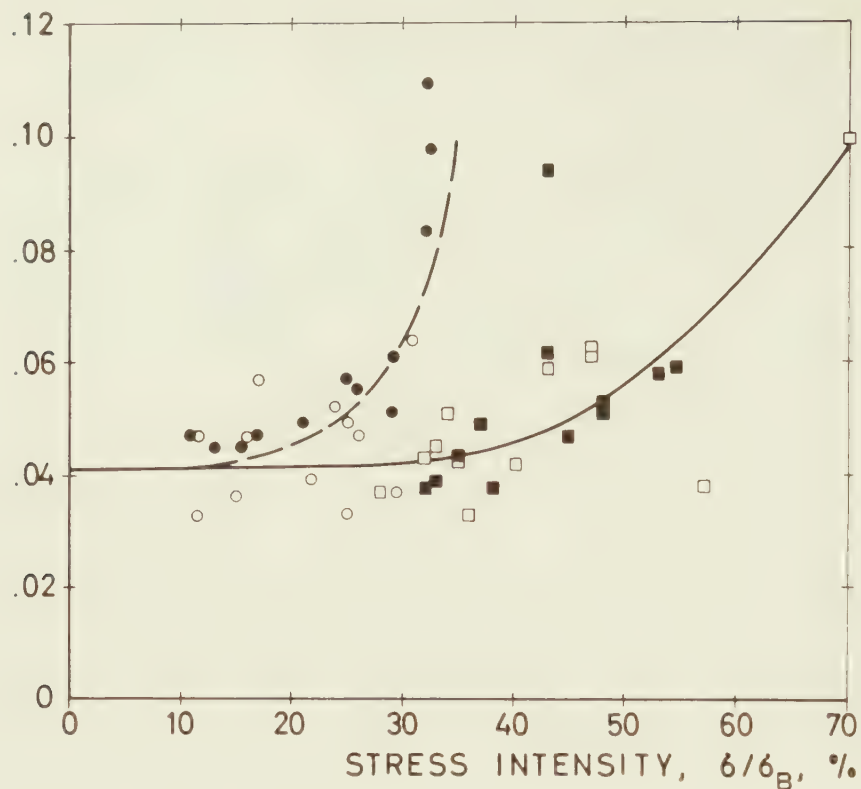
FRACTIONAL CREEP, $\Delta\phi$ (1/256 - 1 day)

FIG. A.4 The fractional creep of the small and the large specimens as function of stress intensity, σ/σ_B .

Symbols:	65% RH	80% RH	Trend
Small	○	●	— — —
Large	□	■	—————

FRACTIONAL CREEP,

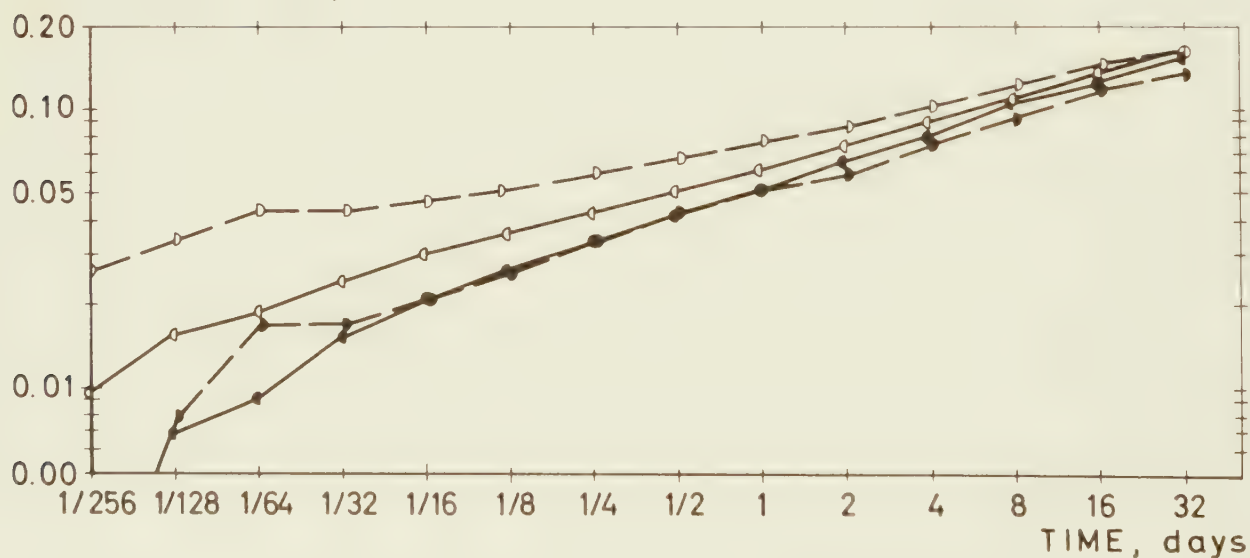


FIG. A.5 Fractional creep vs. time data from *Cederberg and Danielsson (1970)*.

Small specimens, original evaluation	○ — — — ○
Small specimens, "zero" time = 1/256 day	● — — — ●
Large specimens, original evaluation	□ — — — □
Large specimens, "zero" time = 1/256 day	■ — — — ■

The influence of the stress intensity is most clearly seen in FIG. A.4, which shows the fractional creep vs. the stress intensity, σ/σ_B . Small and large samples at both relative humidity levels are given different symbols.

* The fractional creep of the small samples increases significantly more with the stress intensity than that of the large samples.

A reason for this difference might be the influence of the knots. (The fact, that the stress distributions in the two types of measuring devices are not the same, is judged to be of minor importance in this connection.) The existence of knots and other irregularities in the wood influences the magnitude of the fractional creep as well as the magnitude of the fracture stress. The influence on fractional creep is not great, probably because the creep and the initial deformation are influenced to the same extent. However, the influence on the fracture stress is very marked. The large beams may be up to 50% weaker than the small, see TABLE A.1. Hence the reason why the curve for the large samples lies lower than the curve for the small ones is that the σ/σ_B values for the large samples are bigger because of the smaller σ_B .

The curve for the small samples is very steep. Similar curves for wood have been reported by *Sawada* (1957). He also tested small samples (cross section 10 x 10 mm).

I demonstrated in Section 5.3 that the appearance of a creep curve in a log-log plot is greatly influenced by the choice of "zero time". This point will be illustrated by an example.

Two of the creep experiments were extended to 32 days, one with a large and one with a small sample. (No. 46, pine, 65% RH). The results from these two experiments are shown in FIG. A.5. The two upper curves are the original ones from Cederberg and Danielsson. They did not use a consistent zero time to compute $\Delta\Phi$.

TABLE A.1. BASIC CREEP OF WOOD. Experimental results.												
Relative humidity		65%										
Species		Spruce						Pine				
Grade		T 200			T 300			T 200			T 300	
Stress, σ , kp/cm ²		80	160		100	200		80	160		100	200
Sample number		7	26	11	21	20	22	33	44	52	54	60
LARGE	$E_o \cdot 10^{-3}$ kp/cm ²	122	97	118	137	148	119	137	129	103	125	118
	$\Delta\phi(1/256 - 1 \text{ day})$ %	3.3	9.9	4.2	3.7	5.1	3.8	4.2	4.3	6.3	4.5	5.9
	σ_B kp/cm ²	350	230	455	350	590	350	310	495	340	305	420
	σ/σ_B %	36	70	35	28	34	57	40	32	47	33	43
SMALL	$E_o \cdot 10^{-3}$ kp/cm ²	110	148	107	123	159	143	107	142	90	98	106
	$\Delta\phi(1/256 - 1 \text{ day})$ %	3.3	3.9	4.9	3.6	3.3	3.7	4.7	5.7	4.7	4.6	6.4
	σ_B kp/cm ²	695	738	633	667	791	680	725	945	619	625	646
	σ/σ_B %	11.5	21.7	25.2	15.0	25.2	29.4	11.0	16.9	25.8	16.0	31.0

TABLE A.1. BASIC CREEP OF WOOD. Experimental Results. Continued.													
Relative humidity		80%											
Species		Spruce						Pine					
Grade		T 200			T 300			T 200			T 300		
Stress, σ , kp/cm ²		80	160		100	200		80	160		100	200	
Sample number		2	31	24	19	8	28	51	53	61	43	41	56
LARGE	$E_O \cdot 10^{-3}$ kp/cm ²	115	128	140	145	128	125	108	102	91	110	107	120
	$\Delta\Phi(1/256 - 1 \text{ day})\%$	3.8	5.1	3.9	3.8	4.7	5.3	4.9	6.2	9.4	4.3	6.9	6.8
	σ_B kp/cm ²	330	335	490	310	445	415	335	370	370	290	370	380
	σ/σ_B %	38	48	33	32	45	48	37	43	43	35	54	53
SMALL	$E_O \cdot 10^{-3}$ kp/cm ²	118	115	147	128	153	115	119	156	105	110	114	125
	$\Delta\Phi(1/256 - 1 \text{ day})\%$	4.5	6.1	5.7	4.7	5.1	9.8	4.7	4.9	5.5	4.5	8.3	10.9
	σ_B kp/cm ²	628	554	644	585	686	614	708	766	620	639	625	625
	σ/σ_B %	12.7	28.9	24.9	17.1	29.2	32.6	11.3	20.9	25.8	15.7	32.0	32.0

There seems to be a significant difference between them. The two lower curves are based on a consistent zero time of $1/256$ days. Taking into account the normal scatter these two latter curves can be said to coincide, thus demonstrating the importance of using a consistent zero time.

AN EXPERIMENTAL STUDY OF SORPTION CREEP OF WOOD

The investigation of basic creep of wood described in Appendix A was followed up by an investigation of the sorption creep behaviour of wood. I supervised the investigation, which was carried out by *Nilsson and Persson (1970)* as part of their master thesis. After they left the university I continued the experiments and analysed the results. My report is given here.

B.1 Summary of the experiments

The purpose of the experiments was to investigate the creep of wood during moderate desorption and absorption. The experimental variables were (see also TABLES B.1 and B.2):

- * Relative humidity: Specimens preconditioned for several months in 80% RH, then dried in 65% RH and specimens preconditioned for several months in 65% RH, then wetted in 80% RH.
- * Wood species: Large specimens = Pine, Small specimens = Pine and Spruce.
- * Construction grade: T 200 and T 300 according to the Swedish Standard.
- * Stress level: T 200: 80 and 160 kp/cm²
T 300: 100 and 200 kp/cm²
The stress values were 1 and 2 times the allowable design stresses. By mistake, two T 200 beams were loaded with 100 kp/cm², and two T 300 beams with 80 kp/cm².
- * Specimen size: Small, faultfree beams 2 x 2 x 44 cm and large beams (construction grade) 5 x 10 x 300 cm.

The specimens were sampled at random from a lumberyard at the same time as those tested in basic creep (Appendix A). The test set-up and procedures are described in Section A.1 and FIG. A.1.

Both creep and moisture content was measured continuously. Moisture content was measured differently for the two specimen types. For the small specimens dummies of the same size were used. These were weighed and the moisture content calculated relative to the dry weight. For the large specimens the moisture content was measured using an electrical resistance device (type Delmhorst). The pins of this device were insulated except at the tips which were driven into the wood to different depths, 7.5 mm, 15 mm, and 25 mm. FIG. B.9 shows the pin positions, and the electrical connections.

The small specimens reached moisture equilibrium after 10 days. For lack of time, the large beams were only measured up to 18 days which was not long enough to reach moisture equilibrium, (see FIGS. B.6 and B.7).

The small beams were unloaded after 10 days, the large after 18, and the recovery of both was recorded for 1 day. To see the effect of long-term loading four large beams (No. 48, 63, 59, 39) were left loaded and the deflection and moisture contents recorded for 266 days.

B.2 Results

The data are presented in terms of fractional creep. TABLE B.1 shows the fractional creep, $\Delta\phi$, from $1/256$ days to 1 and to 10 days for the small specimens. E_0 determined from the initial deflections and the density, r_{ou} , are also given. TABLE B.2 shows E_0 and $\Delta\phi$ from $1/256$ days to 1, 10, 18, and 266 days for the large specimens.

FIG. B.1 shows $\Delta\phi(1/256 - 10 \text{ days})$ for the small specimens vs. stress. FIG. B.2 shows $\Delta\phi$ from $1/256$ days to 1, 10, 18, and 266 days for the large specimens vs. stress. TABLE B.3 shows the coefficient of variation for $\Delta\phi$, $c\{\Delta\phi\}$. It was estimated by means of the range-method for each stress level, and the mean of these values are reported. The calculation was made

using $\Delta\phi$ values quoted in TABLES B.1 and B.2

B.3 Conclusions concerning species, grade and stress

FIGS. B.1 and B.2 show that the scatter is very large. The fractional creep tends to increase with stress for desorption (80 $\rightarrow$ 65% RH) for the small samples, but considering all the data, the conclusion must be drawn.

- * The stress level does not have a significant influence on the fractional creep in this experiment.

TABLE B.2 and FIG. B.2 show that grade T 300 beams have the highest $\Delta\phi$ values during desorption (80 $\rightarrow$ 65% RH). Since there is no reason for any interaction between climate and grade to exist the following conclusion may be drawn:

- * The grade of the wood does not have a significant influence on the fractional creep.

FIG. B.1 does not show any systematic difference between the spruce and the pine beams. Therefore it must be concluded:

- * The species do not have a significant influence on the fractional creep of the small specimens.

B.4 Specimen size and climate

Section B.3 demonstrated that neither the species, the stress level nor the grade influence the fractional creep. Mean fractional creep vs. time curves for all the specimens of each beam size and each climate can therefore be calculated and compared.

The mean curves for desorption (80 $\rightarrow$ 65% RH) are shown in FIG. B.3. The curves shall be compared with a straight line for basic creep. They are drawn through the same starting point $\phi = 0.03$ at $1/256$ day. This value is close to the mean of the fractional creep at $1/256$ day for all the small specimens. A power function with $b = 0.19$ is also drawn through

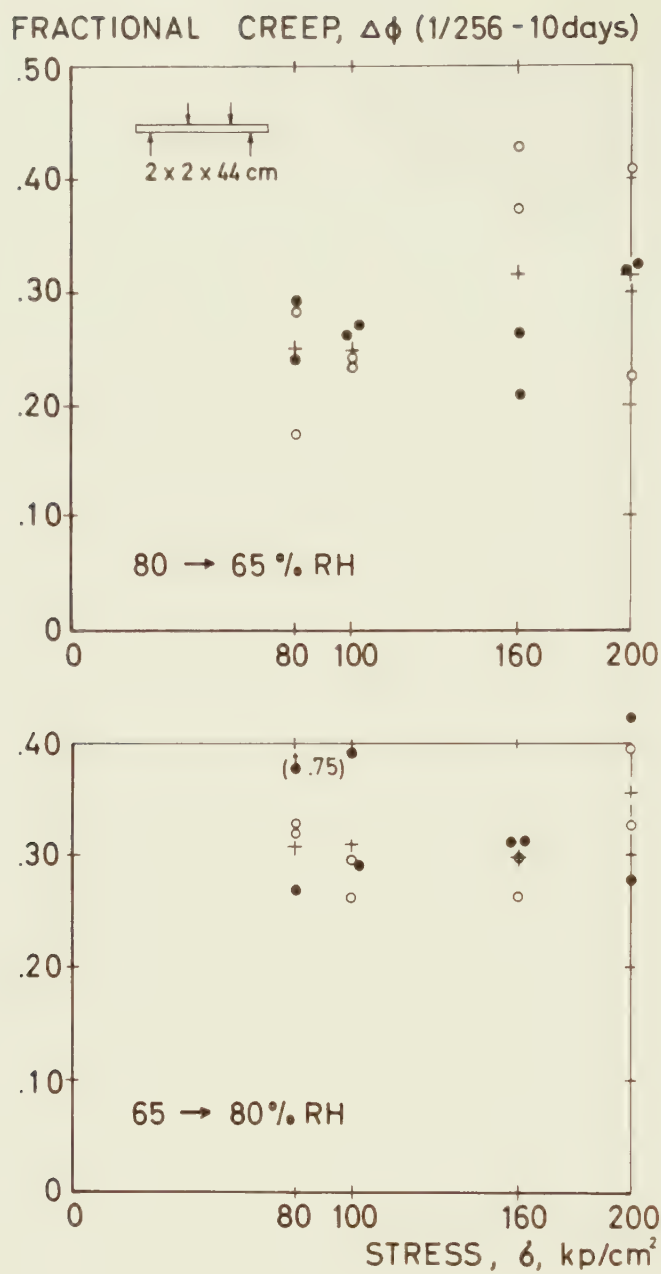


FIG. B.1 Fractional creep, $\Delta\phi$ (1/256 - 10 days), of small beams vs. stress, for desorption and absorption. (O = spruce, ● = pine, + = mean values).

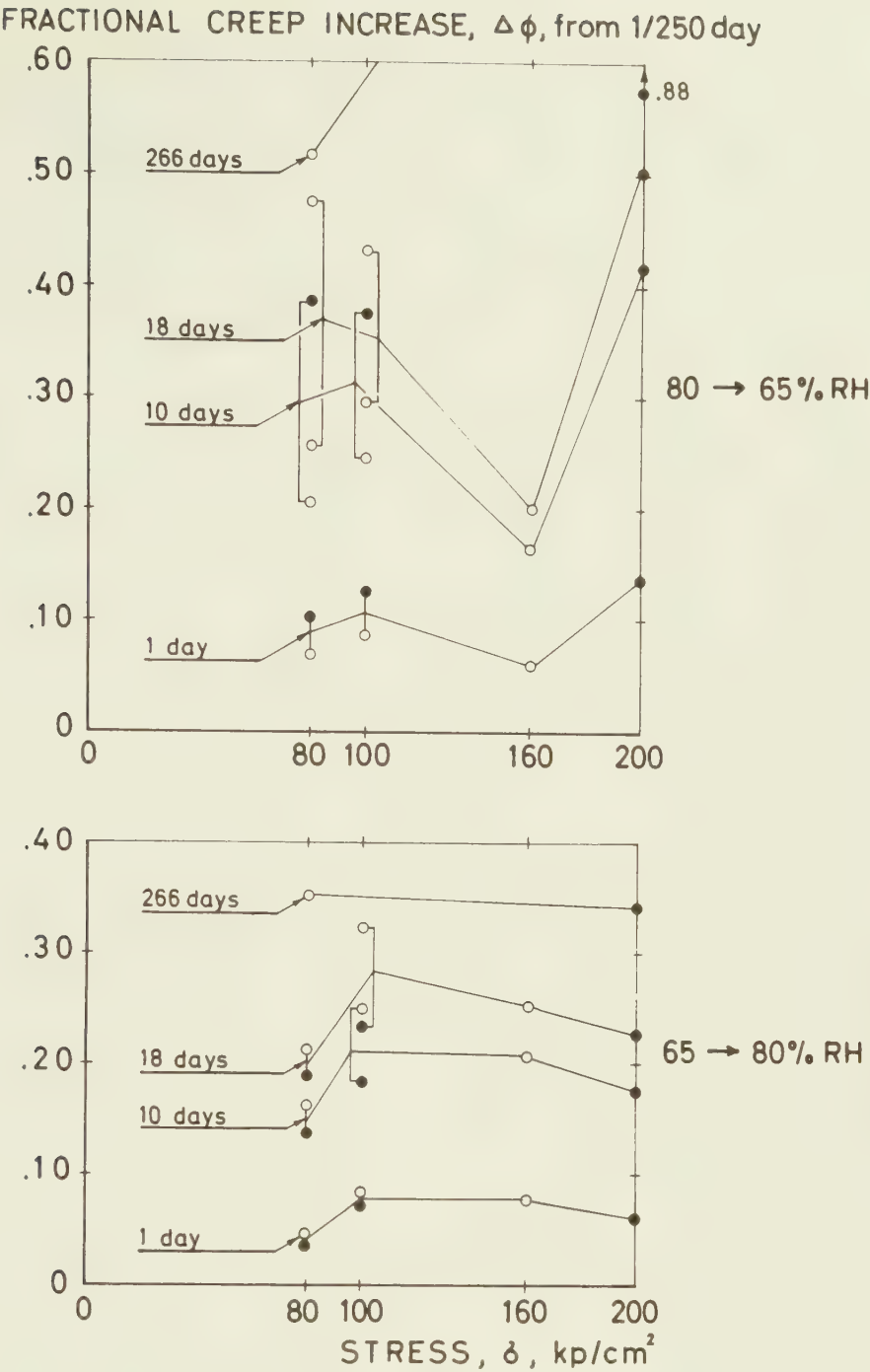


FIG. B.2 Fractional creep from 1/256 days vs. stress for large specimens. The lines connect values recorded at the same times after loading. (O = T 200, ● = T 300).

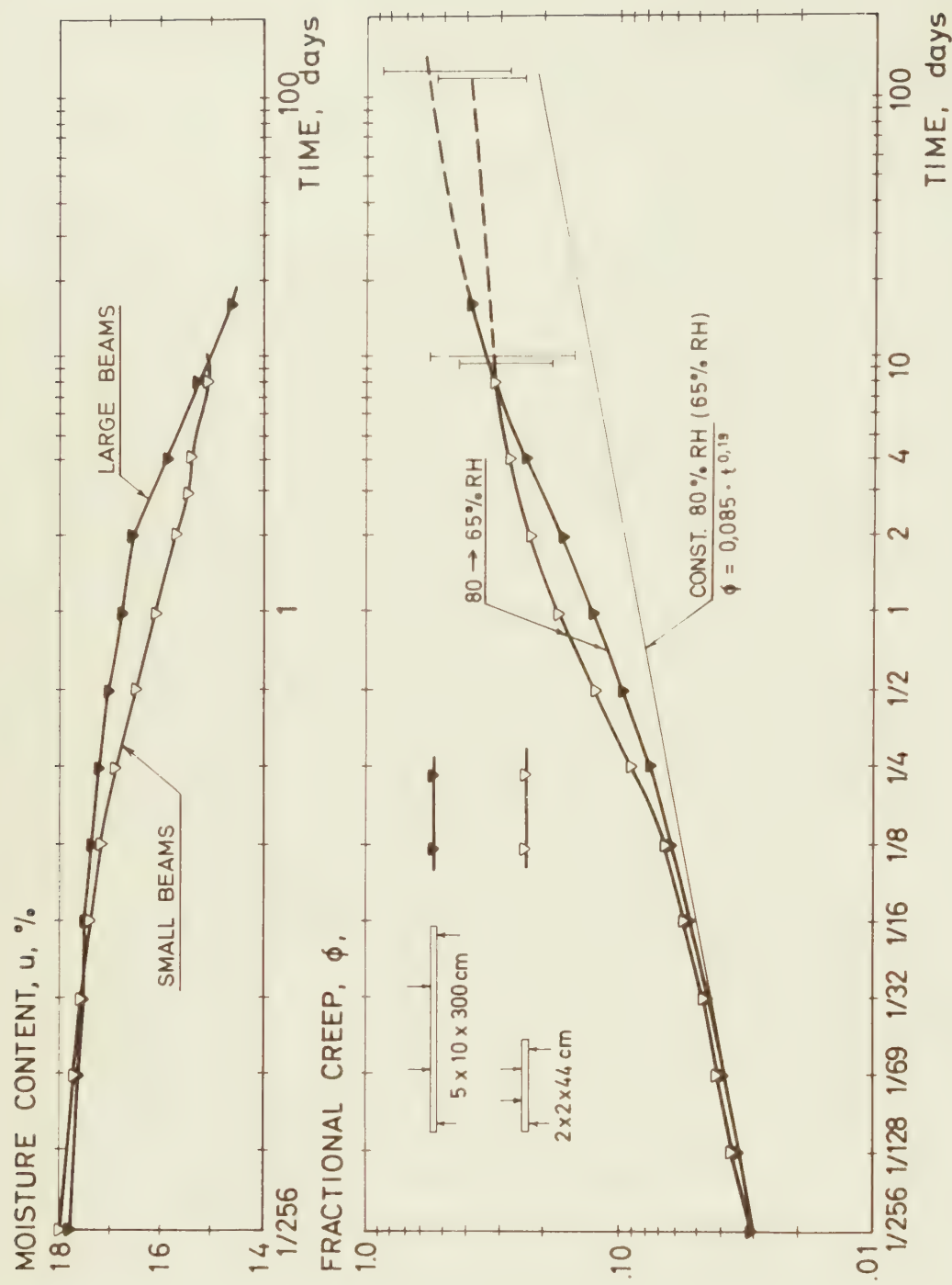


FIG. B.3 Fractional creep vs. time for pine beams during desorption (80% RH $\rightarrow$ 65% RH). Experiments by Nilsson and Persson (1970). Top: Mean moisture content curves. Bottom: Mean fractional creep curves.

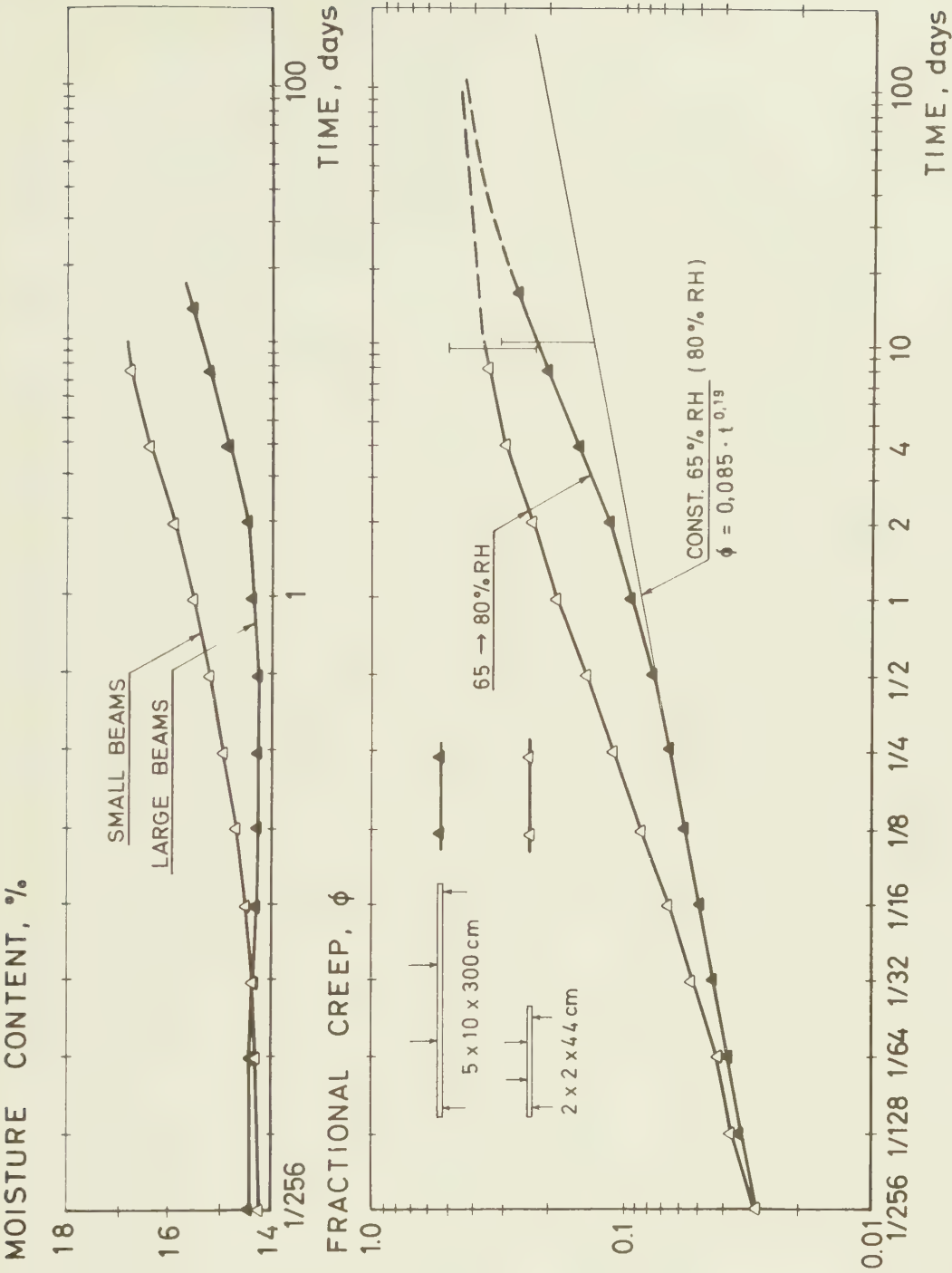


FIG. B.4 Fractional creep vs. time for pine beams during absorption (65% RH $\rightarrow$ 80% RH). Experiments by Nilsson and Persson (1970). Top: Mean moisture content curves. Bottom: Mean fractional creep curves.

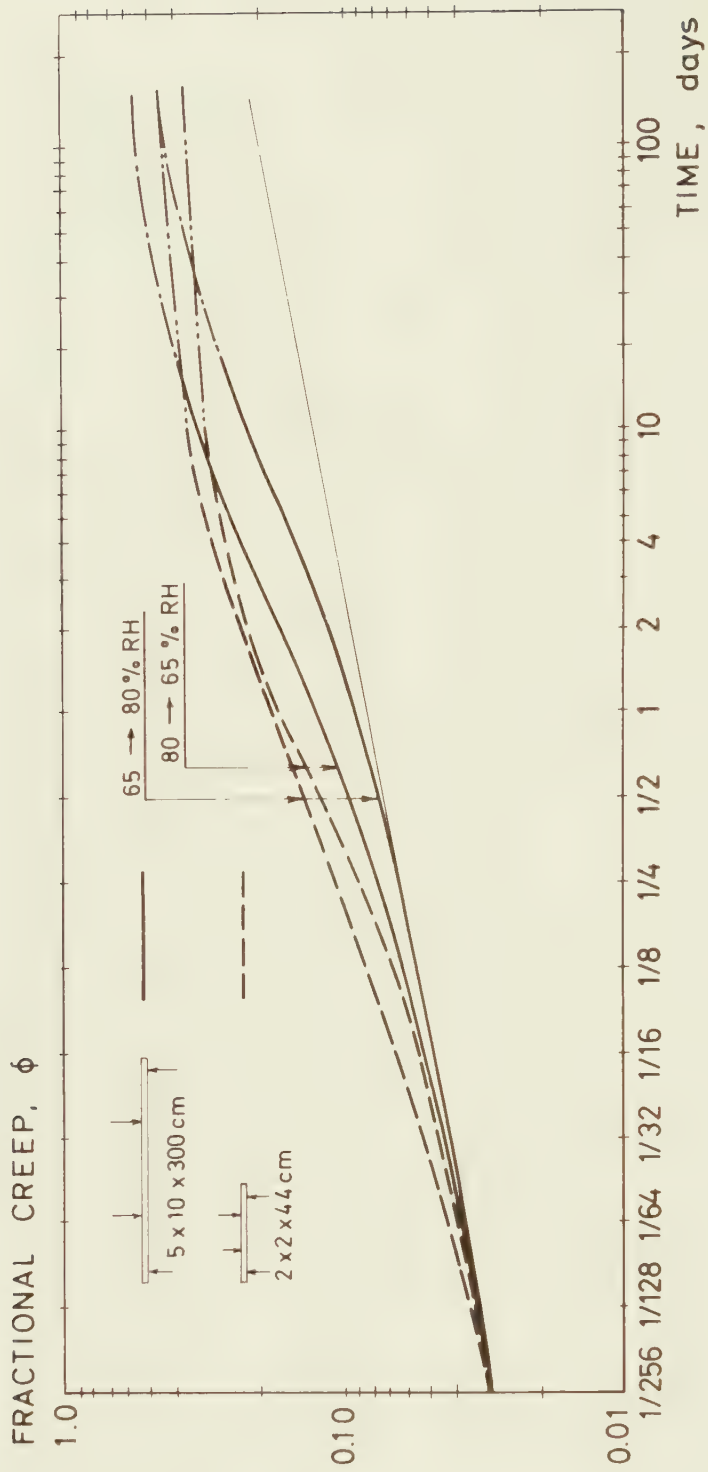


FIG. B.5 Fractional creep vs. time for pine beams during sorption. Mean curves from FIGS. B.4 and B.5 are shown together.

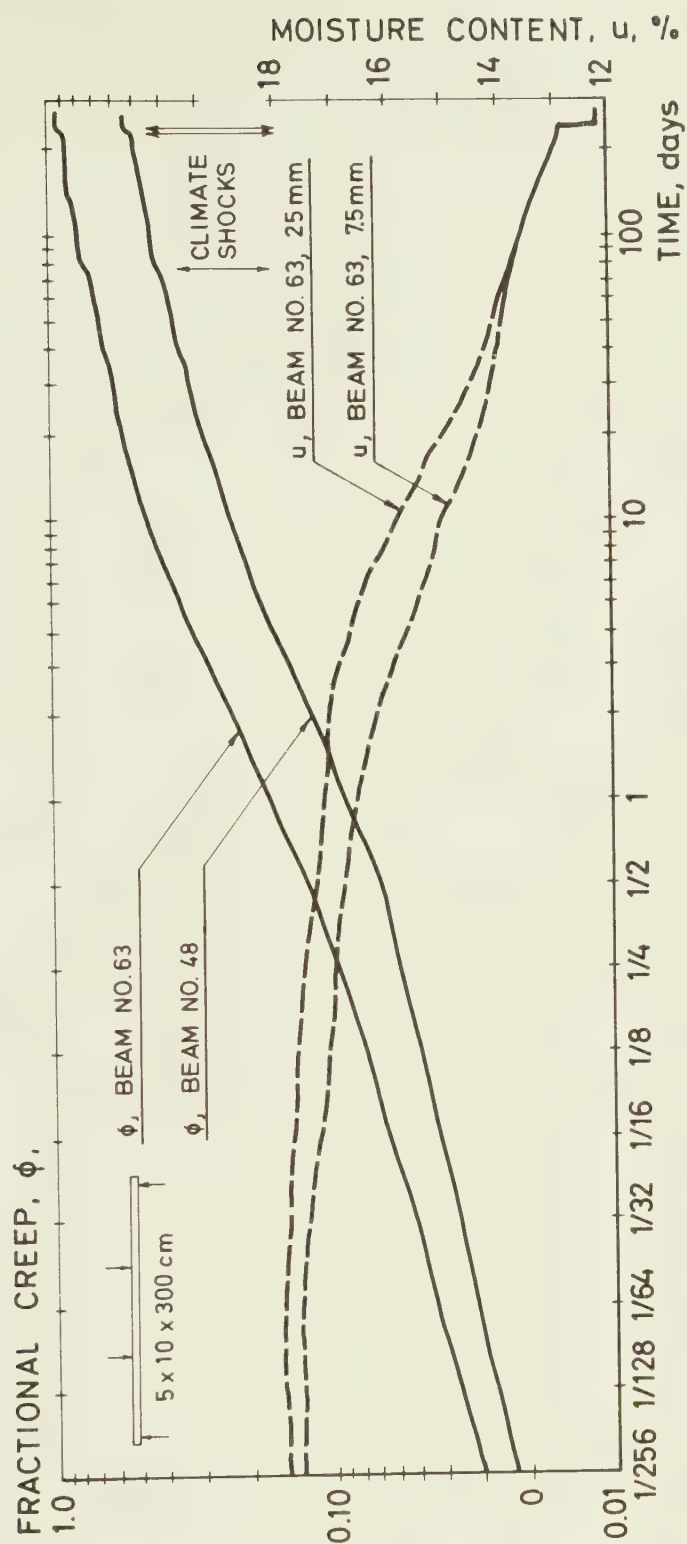


FIG. B.6 Long-term fractional creep of two pine beams during desorption (80% RH $\rightarrow$ 65% RH) (Solid lines). Moisture content changes (Dotted lines).

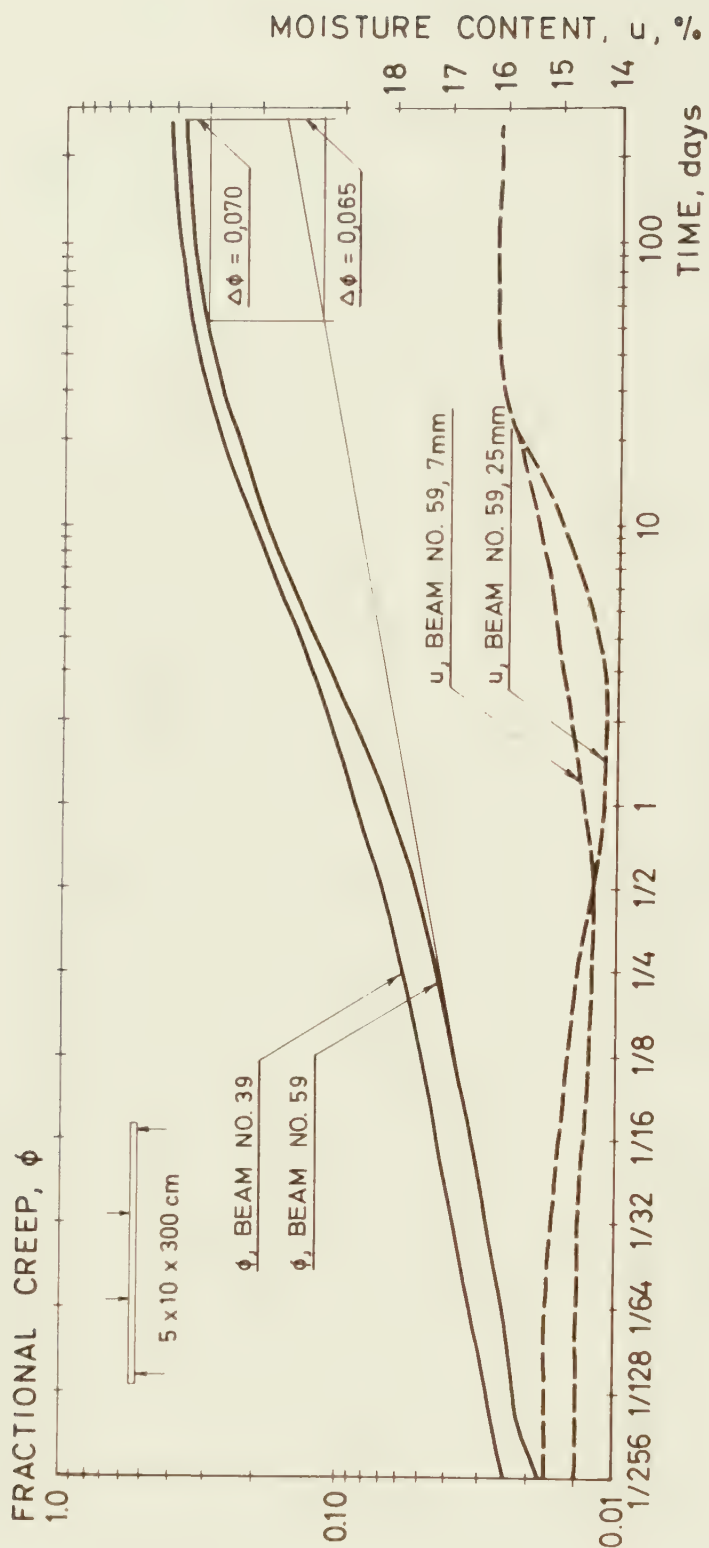


FIG. B.7 Long-term fractional creep of two pine beams during absorption (65% RH $\rightarrow$ 80% RH) (Solid lines). Moisture content changes (Dotted lines).

this point to represent a basic creep curve. The value of $b = 0.19$ is a mean value determined on the basis of the results by Cederberg and Danielsson (Appendix A).

FIG. B.4 shows a similar drawing for adsorption (65 $\rightarrow$ 80% RH). The starting point is the same since no significant difference was found between the fractional creep values at 1/256 day in the two climates.

The four mean curves are drawn together in FIG. B.5 (FIG. 7.13).

The results from the two long-term loaded beams during desorption (80 $\rightarrow$ 65% RH) are shown in FIG. B.6. FIG. B.7 gives the results from the two long-term loaded beams during absorption (65 $\rightarrow$ 80% RH).

Considering FIGS. B.3 - B.7 the following conclusion can be drawn:

- * Absorption as well as desorption during the time under load increases the fractional creep.

FIG. B.5 also shows the influence of the specimen size: As might be expected the small specimens are affected most by environmental change at short times, the large at long times. It is peculiar that both desorption curves lie between the absorption curves. I have no explanation for this behaviour.

The 95% confidence intervals for the fractional creep values at 10 days are drawn in FIGS. B.3 and B.4. From these figures and from FIG. B.5 it is seen that at 10 days there is no significant difference between the fractional creep of the small specimens in the two climates and that of the big specimens during desorption.

The fractional creep curves at longer times for the two specimen sizes are extrapolated and compared in FIG. B.3 and B.4. The extrapolation for the small specimens is made in such a way that the creep rate after 10 days is the same as the creep rate for a

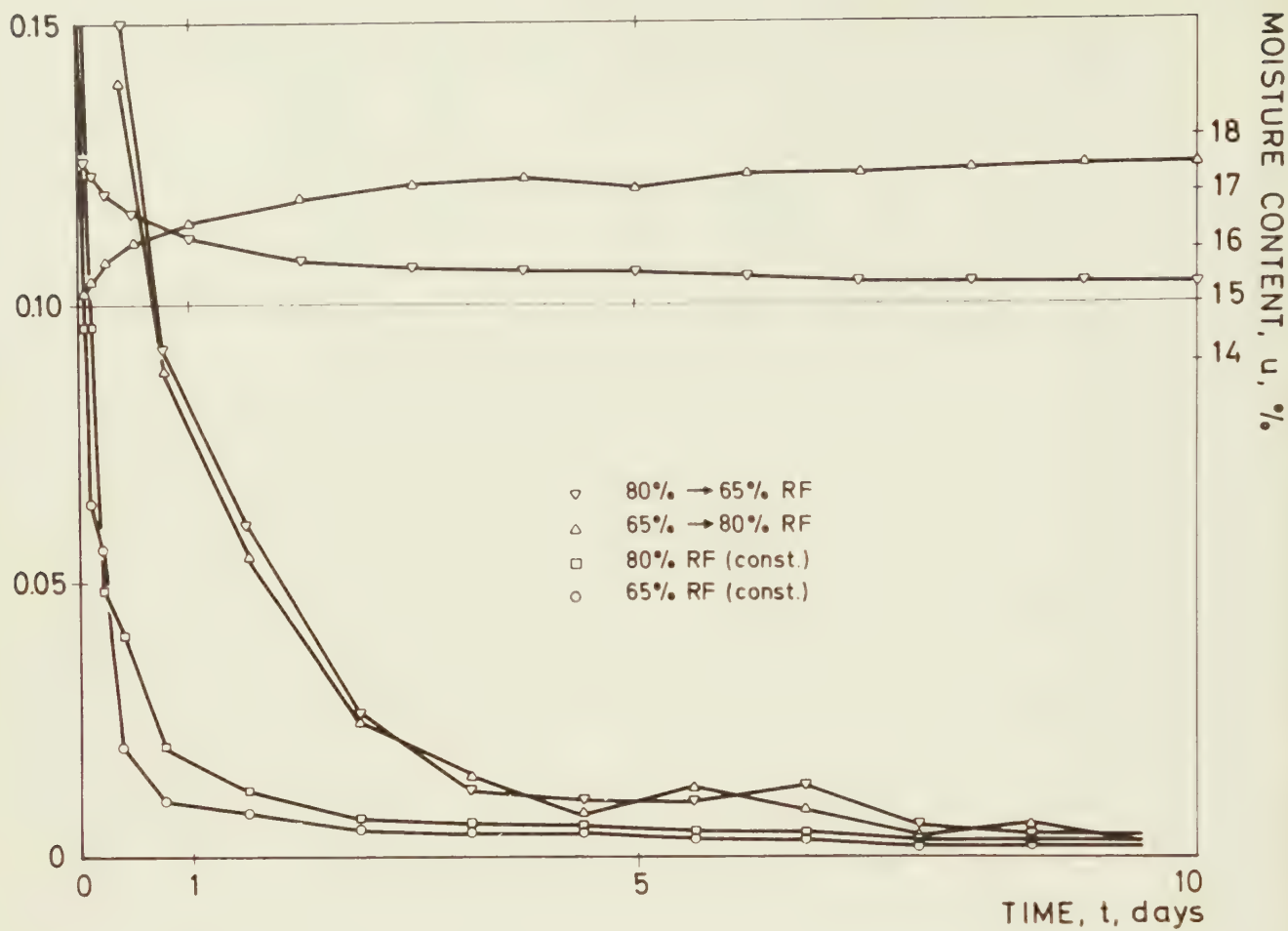
FRACTIONAL CREEP RATE, $\Delta\phi/\Delta t$, (day)⁻¹

FIG. B.8 Bottom: Fractional creep rate vs. time for basic and sorption creep of small spruce beams. Mean values.

Top: Moisture content changes for dummies. Mean values. (*Nilsson and Persson (1970)*).

specimen in constant climate. The curves for the large specimens are made to coincide with those of the long-term experiments shown in FIGS. B.6 and B.7.

The large beams have a greater fractional creep at 200 days during desorption than the small ones, see FIG. B.3. The difference is significant ($p = 0.01$). FIG. B.4 shows that no such difference exists during adsorption.

The fractional creep at 266 days for the large specimens in the two climates can also be compared. Mean values for the extrapolated values in FIGS. B.3 and B.4 and the long-term experiments in FIGS. B.6 and B.7 were calculated. The standard deviation was estimated and the two mean values compared. The number of beams in each climate was 6.

The result was:

Mean in 80 → 65% RH, $\Phi_{266} = 0.64$

Mean in 65 → 80% RH, $\Phi_{266} = 0.43$

Standard deviation $s\{\Phi\} = 0.15$

From this follows:

- * The mean value in the desorption climate (80 → 65% RH) is significantly higher than that in the absorption climate ($p = 0.05$).

The effect of the drying or wetting on creep lasts as long as there is change in the specimen moisture content. Afterwards the creep rate approaches the creep rate of a specimen loaded in a constant climate. This can be seen in FIG. B.8, where the rate of the fractional creep (small specimens) is plotted vs. time. The moisture content changes are shown in the top diagram. The moisture content is constant after 9 or 10 days, and the creep rates for sorption creep is about equal to that of basic creep after 8 days.

This effect can also be seen in FIG. B.7. The fractional creep of specimen No. 59 from 50 days (where the moisture content is constant) to 266 days is

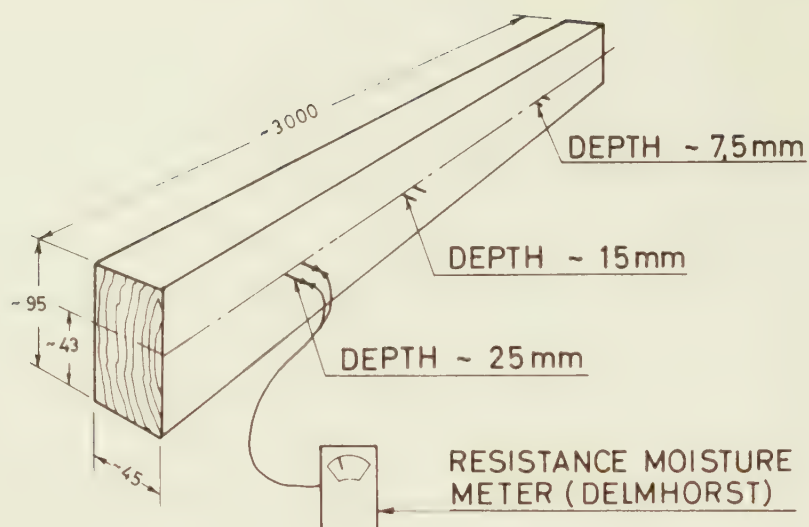


FIG. B.9 The pin positions for moisture content measurements.

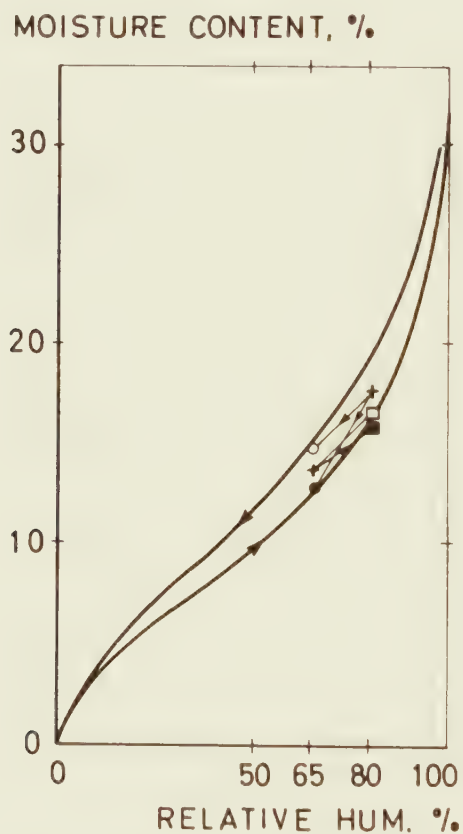


FIG. B.10 Sorption isotherms for spruce obtained by Ahlgren (1971). Mean values for equilibrium moisture contents of large and small beams, obtained by Nielsson and Persson (1970).

	+ Starting points	
	After drying	After wetting
Small beams	○	□
Large beams	●	■

$\Delta\Phi = 0.070$. The corresponding values for the power function $\Phi = 0.057 \cdot t^{0.19}$ is $\Delta\Phi = 0.065$. The agreement between these two values is good. Considering FIG. 7.7 it might be expected, that the value in 80% RH is larger than that in 65% RH as is the case.

B.5 Moisture measurements

As mentioned above the moisture measurements were made in two different ways for the two specimen types, weighing of dummies for the small specimens and resistance measurements for the big ones.

The mean values of the measurements are shown in FIGS. B.3 and B.4. Each of the curves for the small beams is the mean curve of 16 beams. Each point on the curves for the large beams is the mean moisture content at the three depths for 6 beams. The results for the two measurement types do not coincide. The main reason for this is of course that small beams absorb or desorb faster than large beams according to diffusion theory. (The difference between the resistance measurements at 7.5 and 25 mm as seen in FIGS. B.6 and B.7 illustrates the diffusion rate.) Part of the reason may be that the resistance measurements are made on the loaded beam, since it is known that the moisture content of a gel depends on its state of stress (*Barkas (1949)*). It is remarkable that the moisture content decreases during the first day in the climate 65 → 80% RH, when it is expected to increase. This effect is most pronounced in FIG. B.7, especially at a depth of 25 mm.

The equilibrium moisture contents of the loaded beams are also smaller than those of the unloaded dummies. The equilibrium values are:

80 → 65% RH:	Mean of 16 dummies	15%
	Mean of 6 loaded beams	13%
65 → 80% RH:	Mean of 16 dummies	17%
	Mean of 6 loaded beams	16%

These mean values are plotted in FIG. B.10 together with the sorption isotherm for spruce obtained by *Ahlgren (1971)*. It is seen that the values of the dummies coincide well with the sorption isotherms, whereas the values for both loaded beams are lower.

The phenomenon may be explained by the finding of *Barkas (1949)* that a gel in compression has a lower equilibrium moisture content than an unloaded gel. This explanation is only tentative, since the state of stress in the middle of the beam is not known. The experiment was not planned to show this effect. A further investigation of the phenomenon ought to be made.

B.6 Main conclusions

The main conclusions of the investigation are the following:

- * Concerning creep during desorption (80 → 65% RH):
After 10 days the fractional creep is 4 times greater than the basic fractional creep in 80% RH or 65% RH. After about 9 months the fractional creep of the large specimens is significantly greater than the fractional creep of the small specimens.
- * Concerning creep during absorption (65 → 80% RH):
After 10 days the fractional creep of the small beams is 4 times, and that of the large beams 2 times, greater than the basic creep anywhere in the same humidity range. After about 9 months there appears to be no significant difference between the creep of the small and the large specimens.
- * After about 9 months the fractional creep of the large specimens in desorption is significantly greater than that of the same beams in absorption.
- * The fractional creep rate during sorption creep is greater than during basic creep. When the sorption process is completed the rates become equal.

- * Neither the stress, the species nor the grade has any significant influence on the fractional creep.
- * The equilibrium moisture contents of the loaded, large beams are lower than the equilibrium moisture contents of the unloaded small dummies.

TABLE B.1. SORPTION CREEP OF WOOD. Experimental results for small beams														
Change of humidity		80 → 65%												
Species		Spruce												
Grade		T 200					T 300			T 200				
Stress, σ , kp/cm ²		80			160		100		200		80		160	
Sample number $E_o \cdot 10^{-3}$ kp/cm ² Density, r_{ou} g/cm ³ $\Delta\Phi(1/256 - 1 \text{ day}) \%$ $\Delta\Phi(1/256 - 10 \text{ days}) \%$		17a	16a	17b	30a	9a	4a	4b	25a	38a	48a	38b	35b	63a
		134	156	133	139	116	165	127	141	148	145	149	177	135
		0.47	0.42	0.47	0.42	0.41	0.40	0.40	0.39	0.48	0.46	0.48	0.60	0.42
		7.8	16.9	24.3	15.9	13.8	11.8	18.6	10.1	10.2	12.0	12.7	13.1	14.0
		17.5	28.2	42.9	37.3	24.3	23.6	40.7	22.3	29.4	24.0	26.3	20.9	26.3
Change of humidity		65 → 80%												
Species		Spruce												
Grade		T 200					T 300			T 200				
Stress, σ , kp/cm ²		80			160		100		200		80		160	
Sample number $E_o \cdot 10^{-3}$ kp/cm ² Density, r_{ou} g/cm ³ $\Delta\Phi(1/256 - 1 \text{ day}) \%$ $\Delta\Phi(1/256 - 10 \text{ days}) \%$		12a	29a	12b	14a	23a	32a	6a	13a	59a	36a	59b	40a	39b
		153	136	137	141	127	181	136	145	137	111	152	171	179
		0.41	0.43	0.41	0.41	0.36	0.47	0.39	0.38	0.46	0.47	0.46	0.52	0.69
		20.7	18.1	16.5	15.9	18.6	15.4	18.6	22.1	12.1	38.1	17.6	14.6	19.2
		32.0	32.8	29.8	26.3	29.6	26.3	32.5	39.4	27.0	74.6	31.2	31.3	39.0
Change of humidity		Pine												
Grade		T 200					T 300			T 200				
Stress, σ , kp/cm ²		80			160		100		200		80		160	
Sample number $E_o \cdot 10^{-3}$ kp/cm ² Density, r_{ou} g/cm ³ $\Delta\Phi(1/256 - 1 \text{ day}) \%$ $\Delta\Phi(1/256 - 10 \text{ days}) \%$		12a	29a	12b	14a	23a	32a	6a	13a	59a	36a	59b	40a	39b
		153	136	137	141	127	181	136	145	137	111	152	171	179
		0.41	0.43	0.41	0.41	0.36	0.47	0.39	0.38	0.46	0.47	0.46	0.52	0.69
		20.7	18.1	16.5	15.9	18.6	15.4	18.6	22.1	12.1	38.1	17.6	14.6	19.2
		32.0	32.8	29.8	26.3	29.6	26.3	32.5	39.4	27.0	74.6	31.2	31.3	39.0

CREEP OF CELLULAR CONCRETE

This appendix summarizes my investigation of basic creep of unreinforced cellular concrete (aerated concrete, trade names Siporex and Ytong).

C.1 Background

The load and time dependent deformations of cellular concrete have not been thoroughly investigated. This was the conclusion reached in 1962 during the preparation of the Swedish Standards for Building Elements of Cellular Concrete (SBN 1968), which was rather striking, considering the widespread use of cellular concrete in Sweden.

Creep of reinforced elements in bending had been studied earlier (*Nylander and Sahlin (1957)*, *Schäffler (1960)*, *Ödegård (1961)*, *Short (1961)*). Fractional creep data from these investigations are shown in FIG. C.1. The scatter is great, which may reflect the fact that many variables influence the creep (see TABLE C.1).

Few conclusions about the influence of the cellular concrete mass itself on the creep behaviour of the elements could be drawn from the experiments mentioned above. The purpose of my investigation, therefore, was to study the long-term behaviour under load of this type of calcium silicate hydrate.

Most of the experiments were performed under constant moisture conditions (basic creep). Creep, recovery, and creep during re-loading were studied.

C.2 Experiments

FIG. C.2 is a sketch of the test set up. Deflections were measured with a mechanical dialgauge. The material was sealed in 0.1 mm thick polyethylene foil after having been conditioned to the desired moisture content. The load was applied manually. Creep was

TABLE C.1. FACTORS INFLUENCING THE CREEP AND THE DEFLECTION OF REINFORCED ELEMENTS

* Time	* Raw materials and manufacture of cellular concrete
* Stress	
* Temperature	* Arrangements and amount of reinforcement
* Temperature variations	
* Moisture content	* Prestress in reinforcement
* Variations in moisture content	

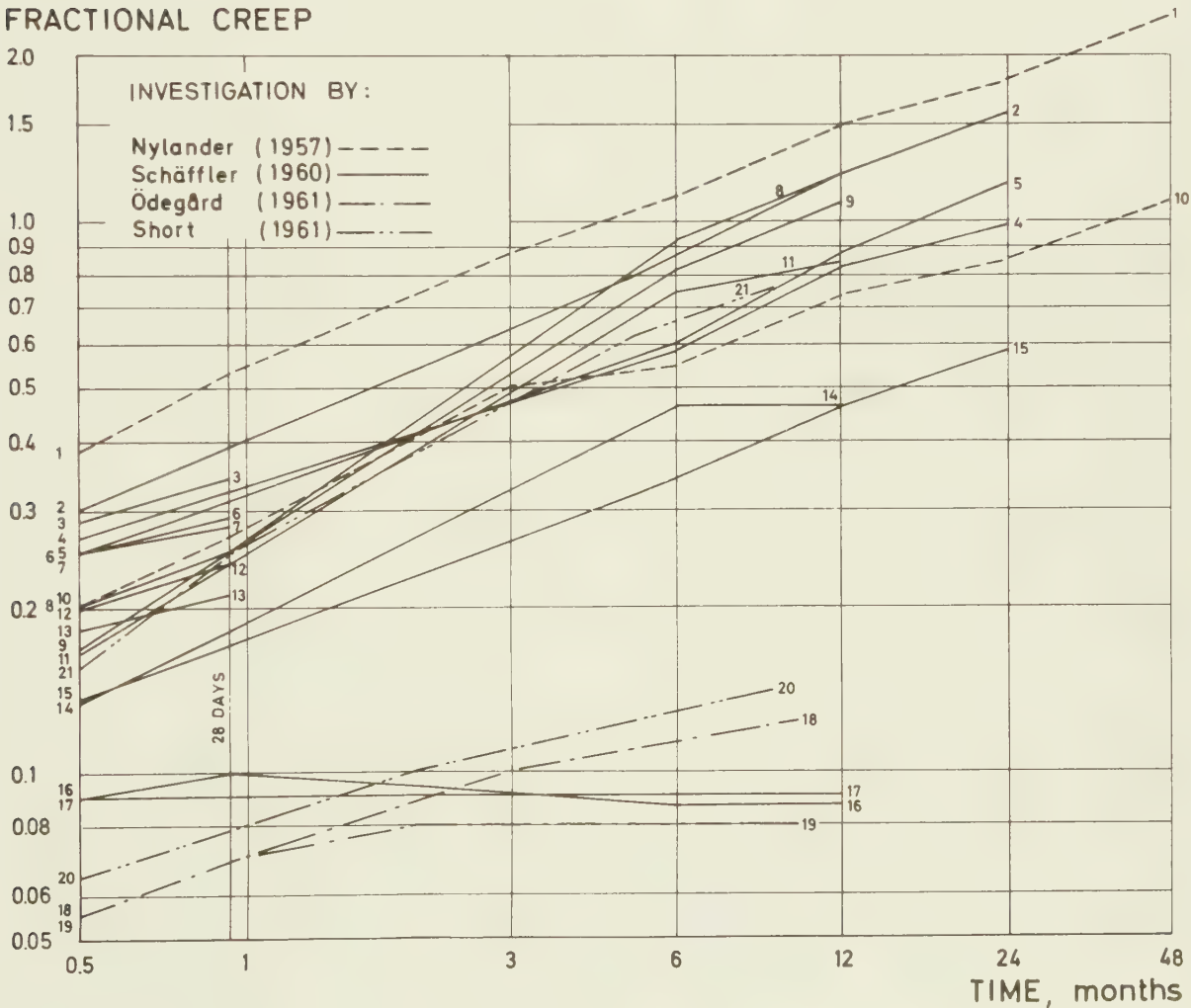


FIG. C.1 Fractional creep vs. time for reinforced elements of cellular concrete in flexure. Previous investigations.

TABLE C.2. CREEP OF CELLULAR CONCRETE
EXPERIMENTAL PROGRAM, MAIN EXPERIMENT

Factory	K				B				S				D			
	Moisture condition **)		d		w		d		w		d		w		d	
Stress level o)	2		4		2		4		2		4		2		4	
	I	II	I	II	I	II	I	II	I	II	I	II	I	II	I	II
Replication	I	II	I	II	I	II	I	II	I	II	I	II	I	II	I	II

**) d = equilibrium at 43% RH, w = watersaturated
o) Stresses in kp/cm²

- Failure

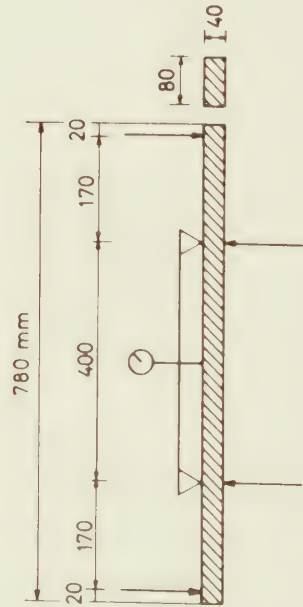


FIG. C.2 Set-up for measuring creep of unreinforced cellular concrete beams.

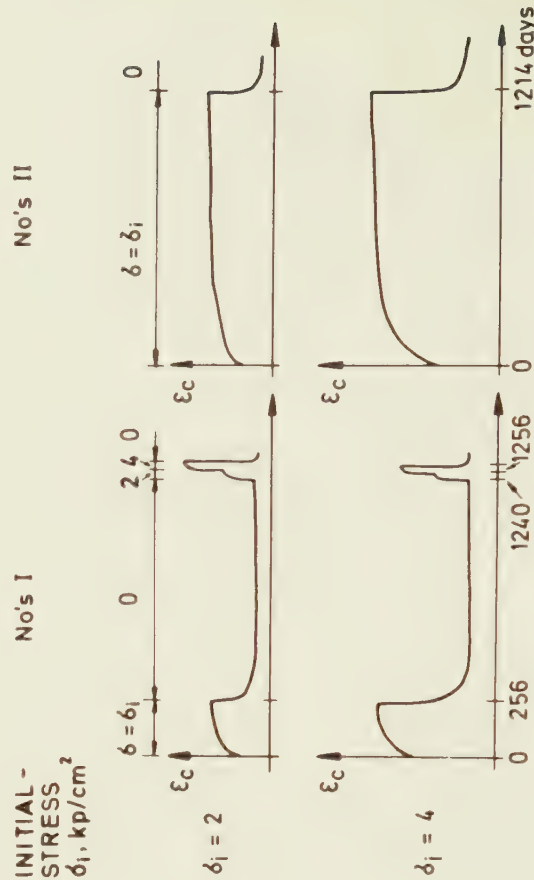


FIG. C.3 Loading and unloading program.

measured from the moment the pointer on the dial stood "still", which was 2 to 4 seconds after loading. (An error in the creep determination is introduced by this method, see Section 5.1. This error may be of the magnitude $\pm 2 \mu\text{S}$.)

A pilot experiment with ten beams from the same factory (D) was carried out, from which the expected scatter in this type of experiments was estimated.

The material used in the main experiment was of nominal density 500 kg/m^3 . It was autoclaved for approximately 10 hours at 180°C , 10 atm. The temperature during the creep experiments was $20 \pm 2^\circ\text{C}$. The experimental program of the main experiment is given in TABLE C.2. Factories, moisture conditions, and stress levels were the variables:

Four factories using quite different raw materials were chosen. K and B are Ytong factories; S and D are Siporex factories.

Two moisture conditions were selected. Equilibrium at 43% RH at 20°C was chosen to simulate room climate in accordance with the Swedish Standard. The water-saturated condition was chosen for its reproducibility, and is taken as representative of the behaviour of the materials at high water contents.

Two levels of flexural stress were selected, 2 and 4 kp/cm^2 .

Two replications were made. These two companion specimens are labelled No. I and No. II in TABLE C.2.

Thus, the main experiment included 32 (31) beams. (One beam from factory D was broken shortly after loading.) These 32 beams of unreinforced material were cut out from 32 reinforced elements randomly sampled from normal production runs. No two beams were cut from the same element.

The loading history of the beams is shown in FIG.C.3. No. I's were unloaded after 256 days and the recovery measured. After 1240 days No. I's were reloaded

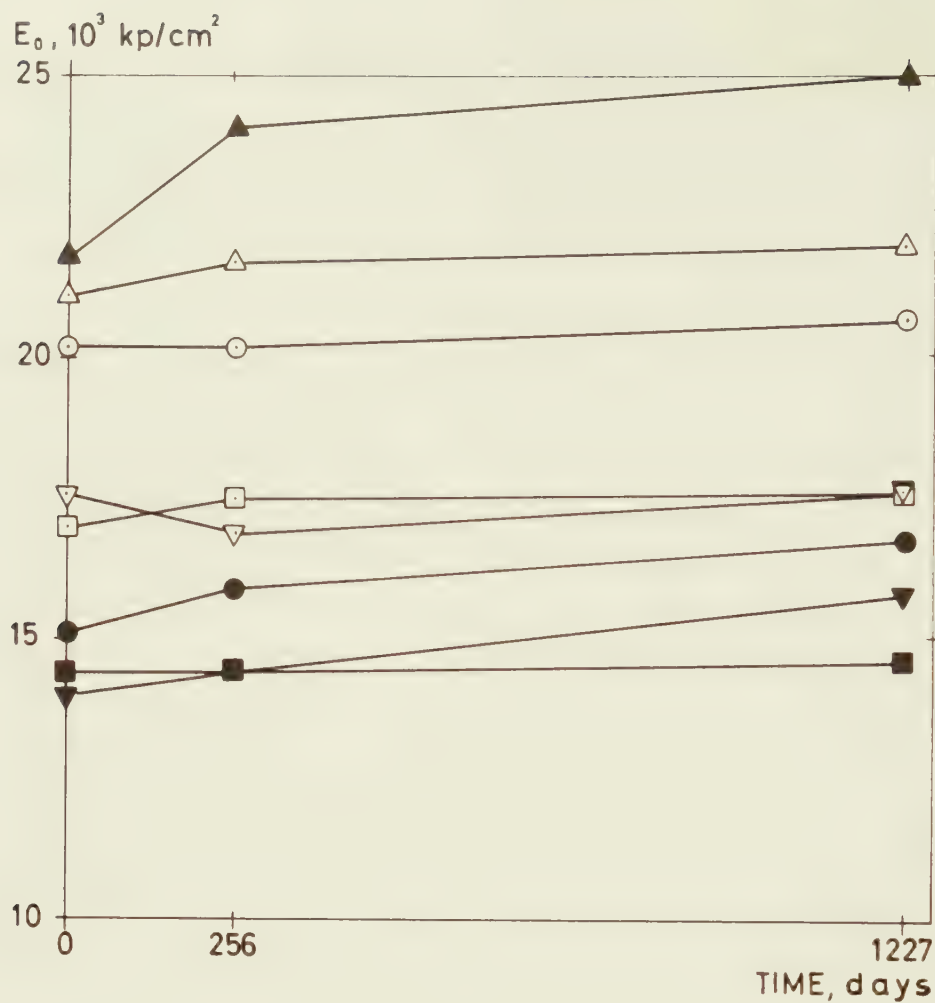


FIG. C.4 Modulus of elasticity, E_0 , vs. time. The points of 0 days are the mean of four values and those at 256 days of two values. The values at 1227 days are the mean of the two values at 1214 and the two values at 1240 days.

Symbols:	Factory:	K	B	S	D
	43% RH:	○	□	△	▽
	Water saturated:	●	■	▲	▼

with 2 kp/cm² for 8 days, then the load was increased to 4 kp/cm² for 8 days, and then the specimens were unloaded. No. II's were unloaded after 1214 days and the recovery measured. E_{dyn} was determined at 1220 days. The design of the experiments and the results for the first 256 days have been reported in detail previously (*Nielsen (1968b)*).

C.3 Results

Initial deformations

The initial deformations upon the different loadings and unloadings can be used to evaluate Young's modulus E_0 . These values are shown in FIG. C.4. E_0 increases with time, especially for the water-saturated beams.

The greatest increase for a single beam is 23%. This increase is probably due to continued hydration and recrystallisation in the calcium-silicate-structure.

It is remarkable that material S has a higher E_0 value in the saturated condition than in the dry condition.

The other materials show the reverse trend, as is common for hygroscopic building materials.

After the last unloading the dynamic modulus of elasticity was determined using flexural vibrations. The values were from 0% to 14% higher than the values of E_0 shown in FIG. C.4 after 1227 days.

Creep

The isochronous stress versus strain diagrams for the first loading are shown in FIG. C.5. The diagrams are constructed by plotting the total strain at some selected times versus the bending stress. The usual stress-strain curve is the curve at $t = 0$. In FIG. C.5 each point up to and including 100 days represents the mean of two companion specimens. The values for 1000 days represent the results for the No. II's. The diagrams show that one can assume

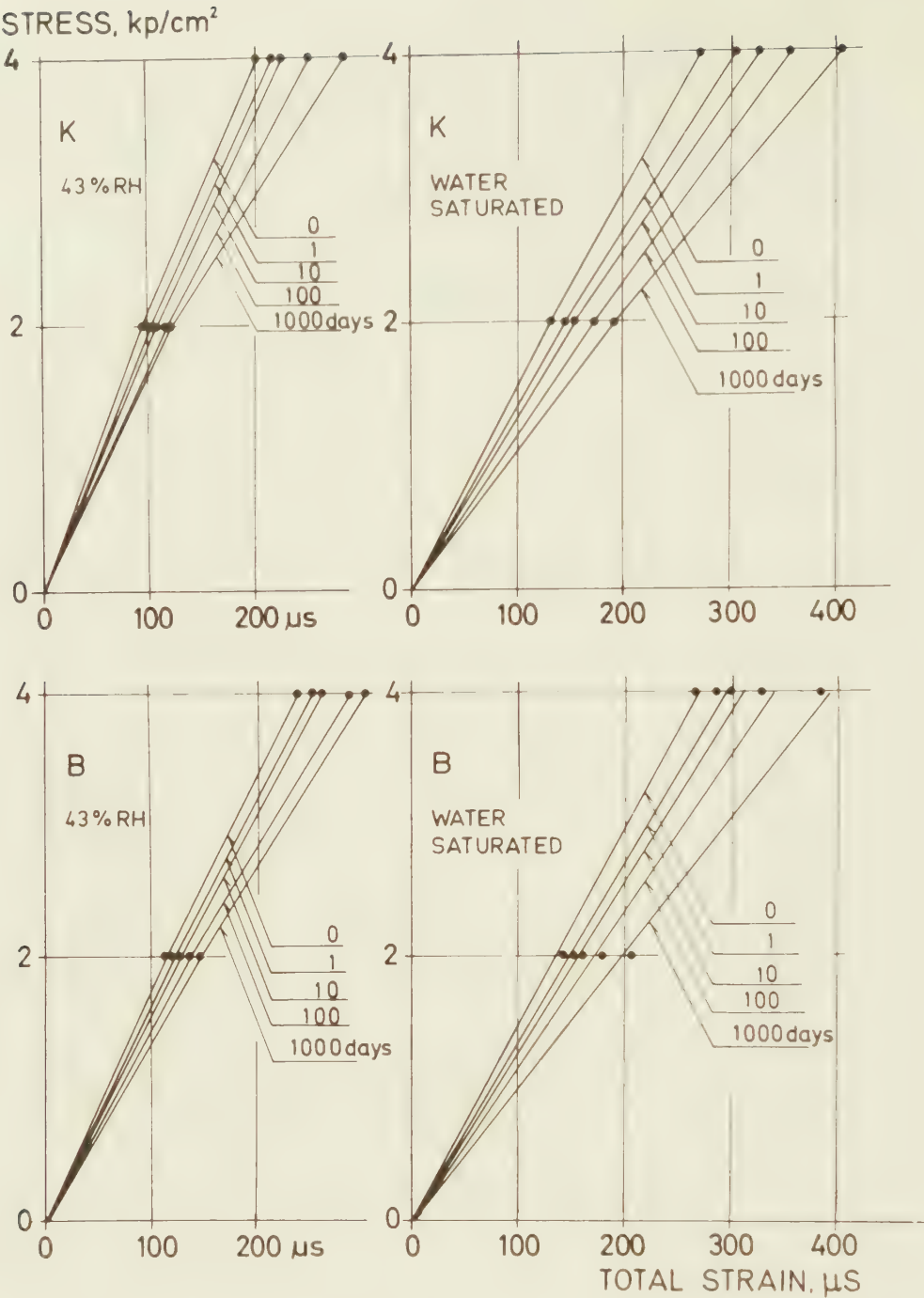


FIG. C.5 (first part). Isochronous stress vs. strain diagrams. The points at each of the two stress levels represent the mean of two experiments.

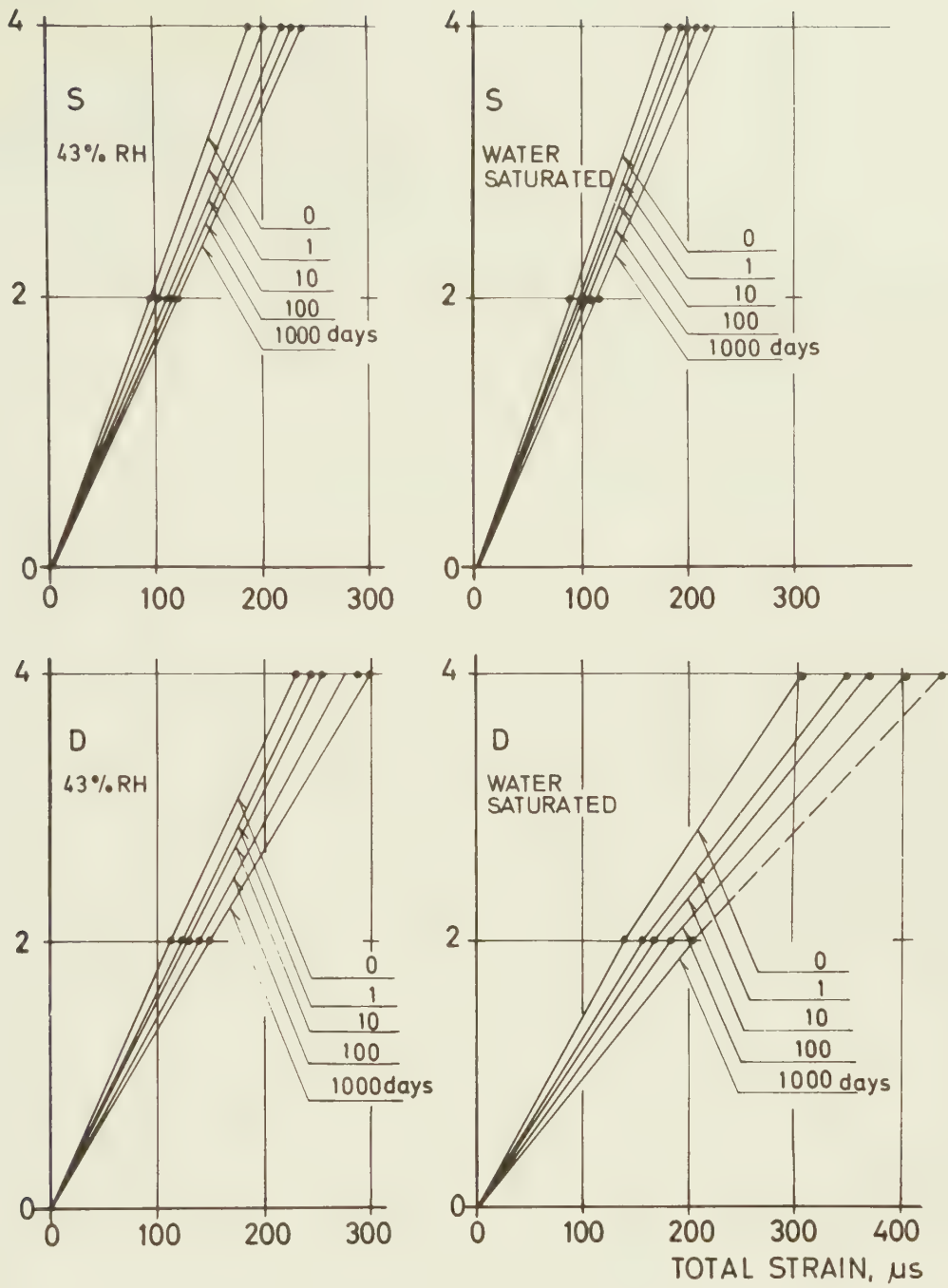
STRESS, kp/cm^2 

FIG. C.5 (continued). Isochronous stress vs. strain diagrams. The points at each of the two stress levels represent the mean of two experiments.

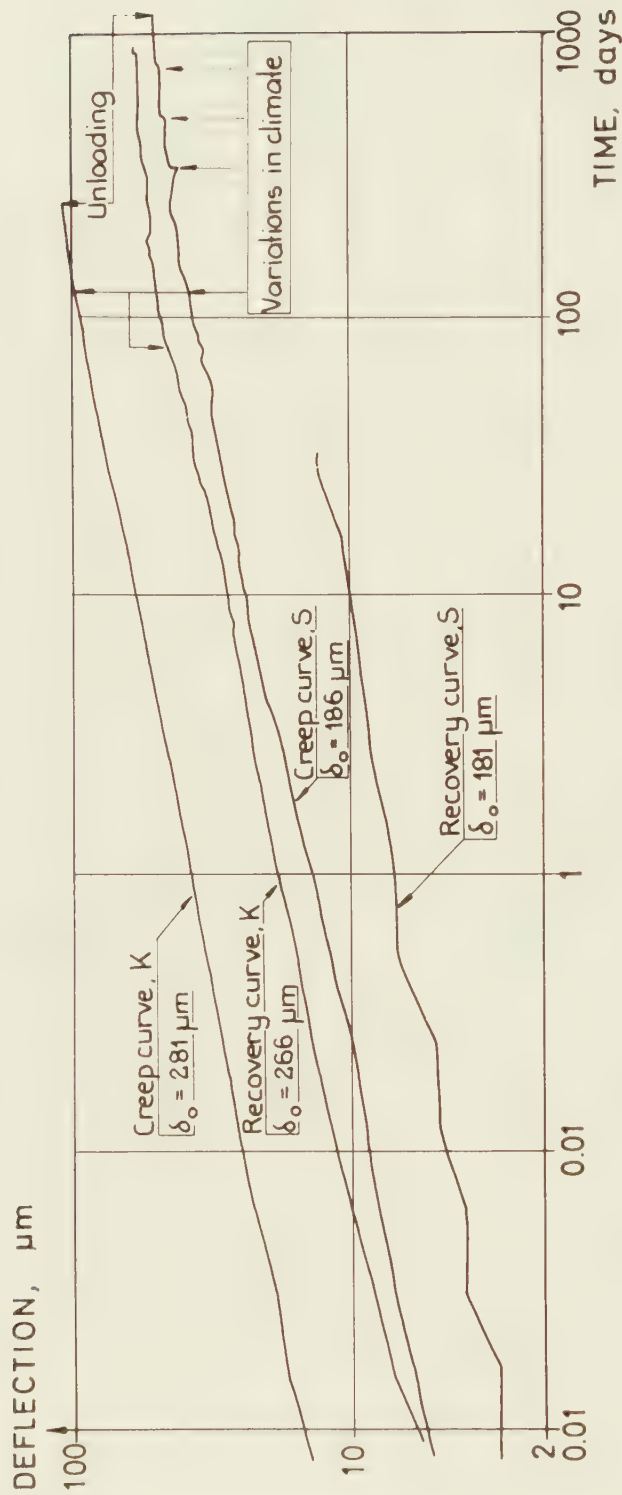


FIG. C.6 Creep and recovery curves for cellular concrete. Factory K, Watersaturated, $\sigma_i = 4 \text{ kp/cm}^2$. Factory S, 43% RH, $\sigma_i = 4 \text{ kp/cm}^2$. δ_0 is the initial deflection.

linearity between stress and strain for this type of experiment and at these stress levels, except for the water-saturated K- and D-materials, where there is a tendency to nonlinearity. This tendency is not statistically significant.

In a bending experiment one is forced to work with rather low stresses. Therefore compression experiments were performed at stresses up to 20 kp/cm². The pilot test showed nonlinearity at stress levels above 4 kp/cm², see FIG. 6.4.

With a constant climate around the test specimen the creep curve can be described by the power function

$$\epsilon_c = A \cdot t^b$$

where A and b are constants. (For linear materials $A = a \cdot \sigma$). This is seen by plotting the creep in a log creep-log time diagram (FIG. C.6). The same trend can be seen plotting the results for reinforced elements (FIG. C.1). The only exception to this rule is the material S, where the raw materials are cement, crushed silicate sand and blastfurnace slag. The creep of this material develops slower than the power function predicts. In FIG. C.1 curves 18 and 19 represent elements from the same factory (S), and curves 16 and 17 are also from the same firm, made with similar raw materials. The influences of factory and moisture content on deformations are also seen in FIG. C.5, and in FIG. 7.5, where the creep per unit stress after the first day is shown for the four factories and the two moisture conditions. It is seen that, in equilibrium with 43% RH, the creep is almost equal for the four factories, but in the saturated condition they differ substantially. Factories K and D show twice as much creep, and B 50% more, while S shows the same creep in saturated condition as in the "dry" state. (These effects of factory and moisture content were tested using variance analysis, and were found to be significant

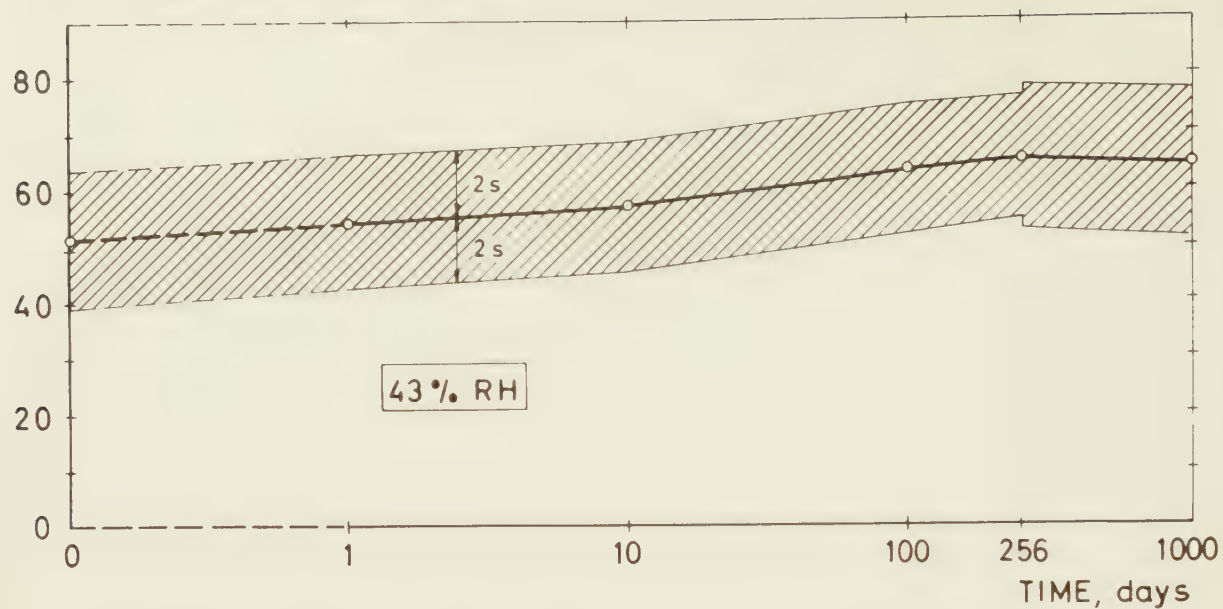
CREEP COMPLIANCE, $\mu S (kp/cm^2)^{-1}$ 

FIG. C.7 Creep compliance vs. time. Mean values for all the specimens in 43% RH. The double residual standard deviation for all the specimens is plotted on both sides of the mean curve.

($p < 0.01$)). What has been said about the influences of the different factors on creep is also valid for the creep compliances.

The behaviour of factory S is remarkable since for other cementitious materials, as well as for other hygroscopic building materials, one usually observes greater creep and total deformations at higher moisture contents than at lower. The reason for this anomalous behaviour must be sought in the microstructure of the material, but it is outside the scope of this investigation to follow that trail.

The analysis of variance mentioned above was carried out for the creep compliances and for the specific creep strains, at the times 1, 10, 100, 256, and 1000 days. The conclusions mentioned above are valid at all these times.

One type of analysis of variance was made within factories and moisture contents and between stress levels. This analysis gives a residual variance, caused mainly by differences in strain response of the replications. Thus, because of the sampling technique, these variances correspond to the variances of creep compliance and specific creep within a random supply of material from the factory studied. There was no significant difference between the residual variances calculated for each of the four factories. Thus, the mean values of the residual variances within the two moisture levels over the four factories were calculated. The square roots of these residual variances give the means of the residual standard deviations.

FIG. C.7 illustrates the relationship between the mean values of the creep compliances and the mean residual standard deviations. The graph refers to the 43% RH level. For each time the mean values, m , for all four factories are plotted together with the values $m \pm 2s$, where s is the mean residual standard deviation for all four factories. The interval

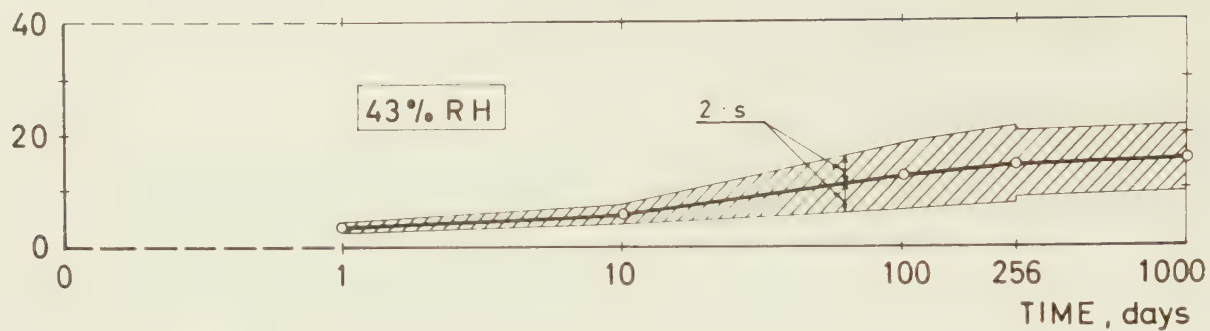
SPECIFIC CREEP STRAIN, ΔJ , $\mu S (kp/cm^2)^{-1}$ 

FIG. C.8 Specific creep strain vs. time. Mean values for all the specimens in 43% RH. The double residual standard deviation for all the specimens is plotted on both sides of the mean curve.

which arises hereby can be taken as a measure of the range within which most of the creep compliances (ca. 95%) of a random supply of materials from one factory will fall. It is seen that the scatter is rather great, but almost independent of time. The jump in the interval at 256 days is caused by the reduction in the number of specimens. (All No. I's were unloaded at 256 days.)

FIG. C.8 is a diagram similar to FIG. C.7, but showing specific strains.

All the above mentioned results are obtained under conditions of constant climate. It is known from tests on other hygroscopic materials that sorption during creep can change the creep behaviour drastically. This was confirmed by a pilot test where watersaturated beams were dried while under load (FIG. 7.12). This test is shown only as an example. To predict creep during drying or wetting is very complicated. The influence of sorption is also illustrated in FIG. C.6. The "jumps" in the creep curve S are caused by climate variations. Most of the values of fractional creep of reinforced elements which are shown in FIG. C.1 are much greater than those found in the creep tests at moisture equilibrium reported here. One of the reasons is certainly that the climate has not been constant during the experiments.

Recovery

The load was removed from No. I's after 256 days and from the others (No. II's) after 1214 days (FIG. C.3). The recovery curves can also be described by a potential function since they are straight lines in the $\log \epsilon_c$ vs. $\log t$ diagram (FIG. C.6). The amount of recovery is about the same for No. I's and No. II's, which means that only permanent creep took place between 256 days and 1214 days (see Section 6.2 and FIG. 6.9). It is remarkable that the recovery continues even about 2 years after unloading.

Re-loading

The re-loading after 1240 days of specimens No. I's, which were unloaded after 256 days, produces less creep than during the first loading, but it is of about the same magnitude as the first measured recovery.

The two-step loading and the unloading show that only negligible permanent strain is developed during this last loading treatment. This means that the first loading has given the material a mechanical stabilization also called strain hardening.

APPENDIX D

CREEP AND RELAXATION

Mathematical relationships between creep and relaxation can be established. An approximate relationship has been derived by *Hansen (1964)* and is given here.

Hansen's notation is the following

- σ_t = the stress at time t ,
- ϵ_{ct} = the creep strain at time t ,
- σ_o = the constant stress during creep,
- ϵ_o = the initial strain produced by the stress σ_o .

He assumed that the creep strain is small compared to the initial elastic strain, and that the creep obtained under decreasing stress can be approximated by the creep that the material would have obtained under the average stress from zero time to the time t .

Using these assumptions the following relationship between creep and relaxation was derived:

$$\sigma_t = \frac{2\epsilon_o - \epsilon_{ct}}{2\epsilon_o + \epsilon_{ct}} \sigma_o \quad (D.1)$$

When fractional creep data are to be transformed to fractional relaxation data, it is practical to rewrite the equation:

$$\frac{\sigma_t}{\sigma_o} = \frac{2 - \frac{\epsilon_{ct}}{\epsilon_o}}{2 + \frac{\epsilon_{ct}}{\epsilon_o}} \quad (D.2)$$

When relaxation data are to be transformed to creep data the following form is practical:

$$\epsilon_{ct} = \frac{2(\sigma_o - \sigma_t)}{\sigma_o + \sigma_t} \epsilon_o \quad (D.3)$$

Or in terms of fractional relaxation data:

$$\frac{\epsilon_{ct}}{\epsilon_o} = \Phi_t = \frac{2(1 - \frac{\sigma_t}{\sigma_o})}{1 + \frac{\sigma_t}{\sigma_o}} \quad (D.4)$$

In his paper Hansen shows the applicability of his formulas to creep and corresponding relaxation data from experiments found in literature on concrete, steel wire, copper, an aluminium alloy, poly(methyl-methacrylate) and polystyrene. In the present report the method is used on wood data (Section 7.2 and 7.3).

Hansen discusses the usage of the method the following way: "The method can be used for any material provided that the creep-stress relationship of the material is not markedly nonlinear over the range of stress met with during stress relaxation. In general, the method yields the best results when creep strain is small compared to elastic strain under the same stress. Because of these limitations, the method is usually not applicable at stress levels approaching creep rupture."

If the assumptions mentioned above are not fulfilled, one may use other formulas. An extensive mathematical literature is available on this subject, (*L.E. Nielsen (1965), Ferry (1968), Chu (1970), Gross (1947)*, the latter rather advanced.).

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